COMPLEX ANALYSIS AND ITS APPLICATIONS

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COMPLEX ANALYSIS AND ITS APPLICATIONS

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COMPLEX ANALYSIS AND ITS APPLICATIONS

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dedicated to the 90th birth anniversary of I.P. Mityuk

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The publication includes plenary lectures and reports of the International Conference *Complex Analysis and its Applications* in the following main areas of research: geometric function theory, holomorphic and quasi-conformal mappings, potential theory, spaces of analytic functions, multidimensional complex analysis, and interrelated issues to analysis.

The conference was held in cooperation with Petrozavodsk State University and Steklov Mathematical Institute of Russian Academy of Sciences supported by the Russian Foundation for Basic Research (grant № 18-01-20023 g).

Intended for researchers, graduate students specializing in the field of complex, real and functional analysis.

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Introduction

The International Conference "Complex Analysis and its Applications", dedicated to the 90th birth anniversary of Professor Igor Petrovich Mityuk, was held from June 2 to 9, 2018 in Gelendzhik on the basis of the branch of the Kuban State University. The conference was organized jointly by the scientists of the Kuban State University, Petrozavodsk State University and the Steklov Institute of Mathematics. The conference was supported by the Russian Foundation for Basic Research (project No. 18-01-20023).

The purpose of this conference is to bring together mathematicians working in the area of complex analysis and its applications. At plenary lectures and invited talks in sections the participants discussed the current state and modern trends in this field. Plenary lectures was focused on modern methods of complex analysis and related fields and are of interest for both junior and senior mathematicians. Invited talks, presented at sections, cover a broad range of applications of various methods of complex analysis to problems in geometric function theory and approximation theory. Several recent results in quasiconformal mappings theory, potential theory, multidimensional complex analysis, functional analysis, theory of partial differential equations, and mathematical physics with applications also was discussed.

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To the 90th birth anniversary of Igor Petrovich Mityuk

(06.01.1928 – 28.09.1995)

Igor Petrovich Mityuk was born in Volchkovo village in Kiev Region on January 6, 1928. Having finished the secondary school as a top student in Penza, he was admitted to the Faculty of Mechanics and Mathematics in Moscow State University without any examinations. His first steps in science were guided by Professor L. A. Lyusternik, a famous mathematician. Due to the fact that Igor Petrovich had a relative who was subjected to repressions in those years, he did not manage to enter post-graduate studies after graduating from Moscow State University in 1950. Instead, he had to work at Maikop Pedagogical Institute as an instructor and then he was appointed Head of the Department. In 1958 Mityuk was admitted to post-graduate school in Kiev Politechnical Institute. His scientific supervisor was Head of a famous school dealing with Geometrical theory of complex variables functions, Professor V.A. Zmorovich. Igor Petrovich's research theme turned out to be very productive. In 1962 he defended his Candidate dissertation and then, in 1966 was awarded Doctor degree. From 1961 to 1963 he worked at the Poltava Engineering and Construction Institute. His successful scientific researches in the field of applications of symmetrization methods to solving extremal problems of geometric theory of complex variable functions were noticed by Academician Yu. A. Mitropolsky, who invited him to work at the Institute of Mathematics of the Academy of Sciences of the Ukrainian SSR. In 1969 he was to return to Kuban by destiny. The Rector of the new Kuban University K.A.Novikov used to know Mityuk very well while working in Maikop, so he offered him to work as Prorector in charge of science. The work in this position opened up new facets of Igor Petrovich's talent. Thanks to his organizational skills, a powerful scientific base was created in the new university, the scientific profile of the university was determined, the university began to train its own scientific personnel. As Scientific Prorector, and then as a dean of the Faculty of Mathematics, I.P. Mityuk did a lot to form and develop the faculty. His high scientific authority, managerial talent and human qualities contributed to form a friendly, creative atmosphere at the faculty.

Mityuk's scientific researches are devoted to studying extremal properties of various kinds of mappings. He was the first in the country to develop new applications of symmetrization methods, enriching the theory of symmetrization with ideas that made it possible to extend fundamental results of univalent functions to the case of holomorphic mapping of multiple connected domains. A general symmetrization principle for multiple connected domains elaborated by him is a powerful tool for researching the properties of distortion and covering in various kinds of analytic functions. New opportunities of using geometrical methods were discovered by I.P.Mityuk in the theory of flat and spatial quasiconformal mappings.

I.P. Mityuk paid a lot of attention to pedagogical activity. His lecture courses on the theory of complex variable functions, special courses and scientific seminars attracted attention of the best students of the faculty of mathematics. The manual written by him on symmetrization methods has been so far a unique textbook, the re-edition of which is planned by the beginning of the conference. Under scientific supervision of Igor Petrovich 11 candidate's theses have been prepared, his students are actively engaged in scientific research. Such well-known scientists in the mathematical world as V.N. Dubinin, A.Yu. Solynin, V.A. Shlyk can be named among them. The achievements of IP Mityuk scientific school are widely known in the country and abroad.

Scientific and organizational talent of IP Mityuk was the key to success of the school-conferences on the geometric theory of functions held under his leadership, with the participation of leading specialists from all over the country. Especially significant were these schools for young mathematicians, who had an opportunity to communicate with famous scientists. Dozens of students and graduate students participating in these schools became candidates and doctors of science.

Plenary Lectures

MSC 30E10 .

Convergence of rational approximants and extremal problems of geometric function theory[1]

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We consider a vector of power series

$$\mathbf{f}(z) := \left\{ f_j(z) := \sum_{k=0}^{\infty} \frac{f_{j,k}}{z^{k+1}} \right\}_{j=1}^{p}$$

which have an analytic continuation along a path in the complex plane that does not intersect a finite set of branch points A :

$$f_j \in \mathcal{A}_A\,, \quad \mathcal{A}_A := \mathcal{A}(\overline{\mathbb{C}}\setminus A)\,, \quad \sharp A < \infty\,.$$

In the series of papers (1978-1984) J. Nuttall put forward a conjecture on the asymptotics of the Hermite–Padé approximants of the vector $\mathbf{f}$. The main ingredient in this conjecture is an algebraic Riemann surface $\mathfrak{R}$ – a $(p+1)$-sheeted covering manifold over $\mathbb{C}$. In terms of the standard functions on $\mathfrak{R}$ the conjecture describes the asymptotics, the domains of convergence of approximants, and the limiting sets of the distribution of zeros of the Hermite-Padé polynomials (see [1]).

For $p = 1$ the Nuttall conjecture states that the diagonal Pade approximants of a function $f \in \mathcal{A}_A$ converge in capacity (of the logarithmic potential) in the "maximal" domain Ω of the meromorphic (single-valued) continuation of f, i.e. the boundary of Ω is the cut of minimal capacity among cuts making f single-valued. This conjecture was proven in 1985 by H. Stahl (even in the more general case: $\mathrm{cap} A = 0$, see [2,3]).

In our talk we discuss motivations, problems and the current progress, see [4,5]) in the proof of the general Nuttall conjecture ($p > 1$).

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MSC 26D15, 26D10.

Conformally invariant integral inequalities and their applications[1]

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Suppose that Ω is a domain of hyperbolic type on the extended plane $\overline{\mathbb{C}}$ of the complex variable $z = x + iy$. Then it is determined the coefficient of the Poincaré metric $\lambda_\Omega(z)$ with Gaussian curvature $\kappa = -4$. Also, we need the definition and some properties of domains with uniformly perfect boundary (see [1]–[5] and the literature therein).

Let $C_0^\infty(\Omega)$ be the family of smooth functions $u : \Omega \to \mathbb{R}$ supported in the domain Ω. We determine and study some special functionals as sharp constants in conformally invariant integral inequalities for test functions $u \in C_0^\infty(\Omega)$. By Δu and ∇u we denote the Laplacian and the gradient, respectively, of the function u.

We will need a numerical parameter $p \in [1, \infty)$. Also, we have to note that the smoothness of $u(z)$ at the point $z = \infty$ means (by definition) the smoothness of $u(1/z)$ at the point $z = 0$.

It is well known that the Dirichlet integral

$$\iint_\Omega |\nabla u|^2 dx\, dy := \iint_\Omega \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 \right] dx\, dy$$

is a conformally invariant quantity. It is easy to show that the integrals

$$\iint_\Omega |u|^p \lambda_\Omega^2(z) dx\, dy, \;\; \iint_\Omega |\nabla u|^p \lambda_\Omega^{2-p}(z) dx\, dy, \;\; \iint_\Omega |\Delta u|^p \lambda_\Omega^{2-2p}(z) dx\, dy \tag{1}$$

are conformally invariant, too. Using these integrals we propose several conformally invariant inequalities for functions $u \in C_0^\infty(\Omega)$. They are similar to some Hardy and Rellich type inequalities. In particular, we examine the following inequalities

$$\iint_\Omega |\nabla u|^p \lambda_\Omega^{2-p}(z) dxdy \geq A_p(\Omega) \iint_\Omega |u|^p \lambda_\Omega^2(z) dxdy \quad \forall u \in C_0^\infty(\Omega), \tag{2}$$

$$\iint_\Omega \frac{|\Delta u|^p}{\lambda_\Omega^{2p-2}(z)} dxdy \geq B_p(\Omega) \iint_\Omega |\nabla u|^p \lambda_\Omega^{2-p}(z) dxdy \quad \forall u \in C_0^\infty(\Omega), \tag{3}$$

$$\iint_\Omega \frac{|\Delta u|^p}{\lambda_\Omega^{2p-2}(z)} dxdy \geq C_p(\Omega) \iint_\Omega |u|^p \lambda_\Omega^2(z) dxdy \quad \forall u \in C_0^\infty(\Omega), \tag{4}$$

where $p \in [1, \infty)$ is a fixed parameter.

We suppose that the constants $A_p(\Omega)$, $B_p(\Omega)$, $C_p(\Omega)$ in these inequalities are defined as maximal constants. More precisely, we determine these constants by formulas

$$A_p(\Omega) = \inf_{u \in C_0^\infty(\Omega), u \not\equiv 0} \frac{\iint_\Omega |\nabla u|^p \, \lambda_\Omega^{2-p}(z) \, dx\, dy}{\iint_\Omega |u|^p \, \lambda_\Omega^2(z) \, dx\, dy},$$

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$$B_p(\Omega) = \inf_{u\in C_0^\infty(\Omega), u\not\equiv 0} \frac{\iint_\Omega |\Delta u|^p \, \lambda_\Omega^{2-2p}(z)\, dx\ dy}{\iint_\Omega |\nabla u|^p \, \lambda_\Omega^{2-p}(z)\, dx\, dy},$$

$$C_p(\Omega) = \inf_{u\in C_0^\infty(\Omega), u\not\equiv 0} \frac{\iint_\Omega |\Delta u|^p \, \lambda_\Omega^{2-2p}(z)\, dx\ dy}{\iint_\Omega |u|^p \, \lambda_\Omega^{2}(z)\, dx\, dy}.$$

According to these definitions, the constants $A_p(\Omega)$, $B_p(\Omega)$ and $C_p(\Omega)$ in inequalities (2)-(4) are non-negative, conformally invariant quantities. In particular, we have the following inequalities

$$0 \le A_p(\Omega) < \infty, \quad 0 \le B_p(\Omega) < \infty, \quad 0 \le C_p(\Omega) < \infty$$

for any domain Ω of hyperbolic type on the extended plane $\overline{\mathbb{C}}$. Notice that the inequality (2) is known (see [2], [3] for the basic case $p = 2$ and see [1], [4] for the general case $p \in [1, \infty)$). In particular, one has the following theorems.

Theorem 1. (see [2], [3]). *If $\Omega \subset \overline{\mathbb{C}}$ is a simply or doubly connected domain of hyperbolic type, then $A_2(\Omega) = 1$.*

Theorem 2. (see [3] for $p = 2$, [4] for $p \in [1, \infty)$). *Let $p \in [1, \infty)$. If $\Omega \subset \overline{\mathbb{C}}$ is a domain with uniformly perfect boundary, then $A_p(\Omega) > 0$.*

Evidently, inequality (2) in a domain Ω is interesting if and only if the constant $A_p(\Omega)$ is a positive number at least for some $p \in [1, \infty)$. But it is known that there exist domains Ω such that $A_p(\Omega) = 0$ for any $p \in [1, \infty)$. For instance, $A_p(\mathbb{C} \setminus \{0, 1\}) = 0$ (see [1] and [3]).

It is to note that the problem to describe geometrically the set of all domains Ω for which $A_2(\Omega) > 0$ is still open (see [3]).

It is clear that one can formulate several open problems with respect to the new constants $B_p(\Omega)$ and $C_p(\Omega)$. We examine some of these problems. In particular, we obtain new results similar to Theorems 1 and 2 for the conformally invariant constants $B_p(\Omega)$ and $C_p(\Omega)$ and discuss some open problems. Also, we will present applications of our results that are connected with isoperimetric inequalities in Mathematical Physics.

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The block element method in applications

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Abstract In the work, the fundamentals of the block element method are outlined. Integral and differential factorization algorithms for construction and use of block elements of the different dimensions are presented. The method is applied to several boundary problems, in particular, to materials with coverings having the breaks. Certain of general properties of the block element method are put forth to show its fairly wide applicability.

Keywords: block element, factorization, topology, integral and differential factorization methods, exterior forms, block structures, boundary problems, seismology

1. Introduction

In the article a vector variant of topological method or research of boundary problem about diverse coverings is developing, which may occur in seismology, materials technology, nanomaterials [1-4]. In particular in seismology this approach allows to estimate intensively disorganized conditions of lithosphere plates, which contain fractures [5]. The case of problem is studied, which was examined in suggestion, that lithosphere plates are exposed only to horizontal balanced influence. As in [1], the plates are fashioned with diverse Kirchhoff plates, their denseness may be disrupted on fractures, (see figure in [1]). Boundary conditions on the plate edges for the case of their horizontal movements may be different, supposed with types of fractures. The approach described in details and used in scalar case provided below, and the main attention is devoted to the description of essentials of its vector prototype. Contrary to scalar case, the vector case needs a realization of differential factorization of index matrix-function of functional equation and Leray residue form [6-8]. Detailed description of vector case algorithm is given. The problems with estimates of protective covering hardness occur in material technology and nanomaterials, these protective coverings either grow old or get defects from environmental factors. The problem of products service with such coverings can be solved by means of pure mathematical analysis of intensively disorganized conditions of such a system [3, 4].

2. Boundary value problem

1. Lets keep all terms for plates, which occur on the third-dimensional base coat, as in [1]. Lets suggest, that their amount is equal to B and every has individual mechanical features. Let's accept coordinate axis x_1ox_2 as resting in the plate surface, and axis x_3 as directing on the outer normal to the base coat. Let's consider the case of horizontal, in the plate surface, harmonic impacts on their surface. Then let us, after cancelling the harmonic fluctuations parameter, present the equations of the marginal sum in the following way

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right) \mathbf{u}_b - \varepsilon_{5b}\mathbf{g}_b = \varepsilon_{5b}\mathbf{t}_b \tag{1}$$

Here every plate is considered as a variety with a margin, where $\mathbf{u}_b = \{u_{1b}, u_{2b}\}$ is a vector of plate points relocation along the horizontal directions of the middle surface. The following denotation must be introduced

$$\mathbf{R}_b\left(\partial x_1, \partial x_2\right) \mathbf{u}_b = \left\| \begin{matrix} (\frac{\partial^2}{\partial x_1^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_2^2} + \varepsilon_{4b})u_{1b} & (\varepsilon_{2b}\frac{\partial^2}{\partial x_1 \partial x_2})u_{2b} \\ (\varepsilon_{2b}\frac{\partial^2}{\partial x_1 \partial x_2})u_{1b} & (\frac{\partial^2}{\partial x_2^2} + \varepsilon_{1b}\frac{\partial^2}{\partial x_1^2} + \varepsilon_{4b})u_{2b} \end{matrix} \right\|$$

The transform of the combined equations differential part by Fourier is given below

$$-\mathbf{R}_b(-i\alpha_1,-i\alpha_2)\mathbf{U}_b=\left\|\begin{matrix}(\alpha_1^2+\varepsilon_{1b}\alpha_2^2-\varepsilon_{4b})U_{1b} & \varepsilon_{2b}\alpha_1\alpha_2U_{2b}\\ \varepsilon_{2b}\alpha_1\alpha_2U_{1b} & (\alpha_2^2+\varepsilon_{1b}\alpha_1^2-\varepsilon_{4b})U_{2b}\end{matrix}\right\|$$

$$\mathbf{U}=\mathbf{F}_2\mathbf{u},\quad \mathbf{G}=\mathbf{F}_2\mathbf{g},\quad b=1,2,...,B$$

$$\varepsilon_{1b}=0{,}5(1-\nu_b),\quad \varepsilon_{2b}=0{,}5(1+\nu_b),\quad \varepsilon_{3b}={h_b^2}/{12},$$
$$\varepsilon_{4b}=\omega^2\rho_b\tfrac{1-\nu_b^2}{E_b},\quad \varepsilon_{5b}=\tfrac{1-\nu_b^2}{E_bh_b},$$

$$g_{1b}=\mu_b(\frac{du_{1b}}{dx_3}+\frac{du_{3b}}{dx_1}),\quad g_{2b}=\mu_b(\frac{du_{2b}}{dx_3}+\frac{du_{3b}}{dx_2}),\quad x_3=0. \tag{2}$$

The following denotations are accepted for the plates: λ, μ are the Lame coefficients, ν is the Poisson coefficient, E is the Young coefficient, h-толщина , ρ - плотность, ω - частота колебаний- $\mathbf{g}_b=\{g_{1b},g_{2b}\}$ and $\mathbf{t}_b=\{t_{1b},t_{2b}\}$ are vectors of contact tensions and external pressures respectively, which occur over the tangent to the base coat border in the Ω_b area. In case of horizontal plate impacts only horizontal constituents of external tensions are left. $\mathbf{F}_2\equiv\mathbf{F}_2(\alpha_1,\alpha_2)$ - two-dimensional and one-dimensional operators of Fourier transforms.

The boundary conditions that are placed on the plate edges are defined by border parts type of each block. Thus, under the accepted denotations, in case the plate edges are toughly squeezed, it's necessary to require that the dislocations in the direction of the local coordinate system axes of the x_1 and x_2 - over the tangent to the middle surface and to the normal respectively are equal to zero, i.e.

$$u_1=0,\quad u_2=0. \tag{4}$$

The normal N_{x_2} and $T_{x_1x_2}$ tangent of the constituents of the middle surface on the plate edge is expressed by the correlations below respectively

$$N_{x_2}=\frac{E}{1-\nu^2}\left(\frac{\partial u_2}{\partial x_2}+\nu\frac{\partial u_1}{\partial x_1}\right),\quad T_{x_1x_2}=\frac{E}{2(1+\nu)}\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right). \tag{5}$$

Different models may be taken as a deformable foundation-base coat (the base-coat where the plate-covers are placed and that is described by a marginal sum).

A deformable half-space, a layer, a multi-layer half-space, including anisotropic one and elastoviscous media may serve as models. In all these cases the correlations between the tensions g_{kb}, $k=1,2,3$ on the stratified medium surface and dislocations u_k, $k=1,2,3$ look like (3) and have the following properties

$$\begin{gathered}\mathbf{u}(x_1,x_2,x_3)=\tfrac{1}{4\pi^2}\int\limits_{-\infty}^{\infty}\int\mathbf{K}(\alpha_1,\alpha_2,x_3)\mathbf{G}(\alpha_1,\alpha_2)e^{-i\langle\boldsymbol{\alpha},x\rangle}d\alpha_1d\alpha_2\\ \langle\boldsymbol{\alpha},x\rangle=\alpha_1x_1+\alpha_2x_2,\qquad \mathbf{K}=\|K_{mn}\|,\quad m,n=1,2,3,\\ \mathbf{K}(\alpha_1,\alpha_2,0)=O(A^{-1}),\quad A=\sqrt{\alpha_1^2+\alpha_2^2}\to\infty.\end{gathered} \tag{6}$$

$K_{ks}(\alpha_1,\alpha_2,x_3)$ - are analytical functions of two complex variables α_1,α_2, meromorphic in particular, numerous examples of them are given in. [9,10]. These correlations are called dominant functions. In case the equations describing the behavior of the foundation medium are known, the matrix elements $\mathbf{K}(\alpha_1,\alpha_2,0)$ may be calculated. If there are no such equations, the dominant functions may be found out experimentally.

3. Block element method

Let us revise the vectorial case of the plate horizontal fluctuations. Then the functional equation of the marginal sum for this case, given for each plate as for a variety with an edge, transforms into a matrix equation of the form [11]

$$\begin{aligned} &-\mathbf{R}_b(-i\alpha_{1b},-i\alpha_{2b})\mathbf{U}_b=\int\limits_{\partial\Omega_b}\boldsymbol{\omega}_b-\varepsilon_{5b}\mathbf{F}_2(\alpha_{1b},\alpha_{2b})(\mathbf{g}_b+\mathbf{t}_b),\\ &\mathbf{U}_b=\{\,U_{1b},U_{2b}\,\}\,,\quad b=1,2,...,B \end{aligned} \qquad (7)$$

Here $\boldsymbol{\omega}_b$ is participated in conception vector of exterior forms, which has such a state $\boldsymbol{\omega}_b=\{\omega_1,\omega_2\}$

$$\omega_{1b}=e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{1b}\tfrac{\partial u_{1b}}{\partial x_2}+\varepsilon_{2b}\tfrac{\partial u_{2b}}{\partial x_1}-i\varepsilon_{1b}\alpha_{2b}u_{1b})dx_1+(\tfrac{\partial u_{1b}}{\partial x_1}-i\alpha_{1b}u_{1b}-i\varepsilon_{2b}\alpha_{2b}u_{2b})dx_2\},$$

$$\omega_{2b}=e^{i\langle\alpha,x\rangle}\{-(\varepsilon_{2b}\tfrac{\partial u_{1b}}{\partial x_1}+\tfrac{\partial u_{2b}}{\partial x_2}-i\alpha_{2b}u_{2b})dx_1+(\varepsilon_{1b}\tfrac{\partial u_{2b}}{\partial x_1}-i\varepsilon_{1b}\alpha_{1b}u_{2b}-i\varepsilon_{2b}\alpha_{2b}u_{1b})dx_2\}.$$

The border of block as it is pointed above may have different characteristics of contact with blocks nearby or may be free. In accordance with algorithm of method of block element [6-8] data concerning nature of contacts should be placed in the form of pseudodifferential equation.

The block element method is enable the get the exect solution of this boundary problem.

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Semigroups of holomorphic mappings of a half-plane and Loewner equation [1]

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The Loewner equation appeared in his famous paper [1] in connection with the description of evolution families in the semigroup of conformal mappings f of the unit disk $\mathbb{D} = \{z \in \mathbb{C}\colon\ |z| < 1\}$ into itself with normalization $f(0) = 0$, $f'(0) > 0$. In connection with the development of its stochastic analogue (Schramm-Loewner Evolution), this equation was called the radial Loewner equation (see, for example, [2]). In the description of evolution families in the semigroup of conformal mappings f of the upper half-plane $\mathbb{U} = \{z \in \mathbb{C}\colon\ \operatorname{Im} z > 0\}$ into itself with hydrodynamic normalization at infinity $f(z) - z \to 0$ as $z \to \infty$, an analogue equation arises [3], which is now called the Loewner chordal equation. The evolution equation in the semigroup of conformal mappings of a strip into itself is naturally called the dipolar Loewner equation.

Our goal is to show that it is most convenient to translate the study of evolution families into a semigroup of holomorphic mappings of the half-plane into itself. The notion of monotone independence in noncommutative probability theory (see [4], [5]) also naturally leads to semigroups of holomorphic mappings of the half-plane into itself. In this connection, the corresponding evolution equations and some noncommutative analogues of the Lévi-Khintchine formula will be considered.

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MSC 30F30, 33E05

Structure of trajectories for a quadratic differential on a three-sheeted Riemann surface of genus 1[1]

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Consider the three-sheeted Riemann surface S of genus 1, corresponding to the algebraic function

$$w = \sqrt[3]{(z-a_1)(z-a_2)(z-a_3)};$$

here a_j are pairwise distinct complex numbers.

There exists an abelian integral G on S which is regular at every point of S, except of points P_0, P_1, and P_2 lying over infinity, with the following asymptotic behavior:

$$G(z) \sim \begin{cases} 2\ln z, & z \to P_0, \\ -\ln z, & z \to P_j, \quad j = 1, 2. \end{cases}$$

We investigate the Nuttall structure of sheets of S, connected with the abelian integral G [1]. This structure is very important for the study of the asymptotics of the rational Hermite–Padé approximants [2].

The Nuttall structure of sheets of S is completely determined by the structure of singular trajectories of a quadratic differential connected with the abelian differential $dG(z)$. In [2] geometric structure of the trajectories is investigated in the case when the triangle Δ with vertices a_1, a_2, and a_3 is sufficiently close to regular one.

We consider the case of arbitrary isosceles triangle Δ and show that, for the quadratic differential mentioned above, there are only two essentially different types of structures of singular trajectories, depending on whether the apex angle of Δ is greater or less than $\pi/3$.

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MSC : 30C45

On Logarithmic Coefficients and some related Conjecture for certain class of Univalent Functions[1]

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Let $\mathcal{A}$ be the class of functions f analytic in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ with the normalization $f(0) = 0 = f'(0) - 1$. Let $\mathcal{S}$ denote the class of functions f from $\mathcal{A}$ that are univalent in $\mathbb{D}$. Then the logarithmic coefficients γ_n of $f \in \mathcal{S}$ are defined by the formula

$$\frac{1}{2}\log\left(\frac{f(z)}{z}\right) = \sum_{n=1}^{\infty} \gamma_n z^n, \quad z \in \mathbb{D}.$$

In this talk, we present an overview on the subject of logarithmic coefficients of certain classes of univalent analytic functions defined on the unit disk $\mathbb{D}$. Our particular emphasize will be to deal with the class $\mathcal{U}(\lambda)$ consisting of functions $f \in \mathcal{A}$ which satisfy the condition $\left|\big(z/f(z)\big)^2 f'(z) - 1\right| < \lambda$ for some $0 < \lambda \leq 1$. For $f \in \mathcal{U}(\lambda)$ with $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, it is conjectured that $|a_n| \leq \sum_{k=0}^{n-1} \lambda^k$ for $n \geq 2$ This conjecture remains open for $n \geq 5$. On the other hand, the authors in [1] prove the following sharp inequality for $f \in \mathcal{U}(\lambda)$:

$$\sum_{n=1}^{\infty} |\gamma_n|^2 \leq \frac{1}{4}\left(\frac{\pi^2}{6} + 2\mathrm{Li}_2(\lambda) + \mathrm{Li}_2(\lambda^2)\right),$$

where Li_2 denotes the dilogarithm function. In this talk, we shall discuss some new results and some new inequalities satisfied by the corresponding logarithmic coefficients of some other subfamilies of $\mathcal{S}$.

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Value regions of univalent functions within different classes [1]

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Around the turn of the last century, the question of where an analytic function defined on the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ or the upper half-plane $\mathbb{H} = \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ can map points $z_0 \in \mathbb{D}$ or $z_0 \in \mathbb{H}$, respectively, was considered for different classes. Recently, numerous publications revived the interest to such problems, see, e.g., [2], [4-9].

The notable progress goes back to an idea by Loewner to express schlicht functions as solutions to a differential equation. Through Loewner's equation, it is possible to interpret an optimization problem for classes of univalent functions as the problem of finding a control that steers the trajectory of a dynamical system to the boundary of its reachable set. The most part of recent results in the general question has been obtained with the help of optimization technique. Powerful tools from the theory of optimal control can be applied to tackle the value region problem expressed via the Loewner equation.

Denote $\mathcal{H}(T)$, $T > 0$, the set of conformal maps from $\mathbb{H} \setminus K(T)$, with arbitrary hulls $K(T)$ of half-plane capacity T, onto $\mathbb{H}$, normalized hydrodynamically as

$$f_K(z) = z + \frac{2T}{z} + O\left(\frac{1}{|z|^2}\right), \quad \mathbb{H} \ni z \to \infty.$$

Roth and Schleissinger [9] found the set $\{f(z_0)\}$, $z_0 \in \mathbb{H}$, for the class $\cup_{T>0}\mathcal{H}(T)$. This research was continued in [7] for the class $\mathcal{H}(T)$ with fixed T. Without loss of generality, assume that $z_0 = i$ and consider the extremal problem to describe the value region

$$D(T) = \{f(i) : f \in \mathcal{H}(T),\ i \notin K(T)\}.$$

To formulate the result for $0 \le T \le \frac{1}{4}$, denote by $C_0(\varphi, T) > 0$, $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$, $0 \le T \le \frac{1}{4}$, the unique root of the equation

$$2\cos^2\varphi \log(1 - \sin\varphi) + (1 - \sin\varphi)^2 = 2\cos^2\varphi \log C + C^2(1 - 4T).$$

For a fixed $T \in (0, \frac{1}{4}]$, this equation has a unique solution $C = C_0(\varphi, T)$ depending on φ.

Theorem 1. *The domain $D(T)$, $0 < T \le \frac{1}{4}$, is bounded by two curves l_1 and l_2 connecting the points i and $i\sqrt{1-4T}$. The curve l_1 in the complex (u, v)-plane is parameterized by the equations*

$$u(T) = \frac{C_0^2(\varphi, T)(4T - 1) + (1 - \sin\varphi)^2}{2C_0(\varphi, T)\cos\varphi}, \quad v(T) = \frac{1 - \sin\varphi}{C_0(\varphi, T)}, \quad -\frac{\pi}{2} < \varphi < \frac{\pi}{2}.$$

The curve l_2 is symmetric to l_1 with respect to the imaginary axis.

Denote by $\varphi_0(T) \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $T > \frac{1}{4}$, the unique solution of the equation

$$\log\frac{1 - \sin\varphi}{1 + \sin\varphi} + \frac{1 - \sin\varphi}{1 + \sin\varphi} + 1 = \log\frac{1}{4T - 1}.$$

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For $T > \frac{1}{4}$ and $\varphi \in [\varphi_0(T), \frac{\pi}{2}]$, denote by $C_0(\varphi, T) > 0$ the minimal root of this equation and by $C_{00}(\varphi, T) > 0$ its maximal root.

Theorem 2. *The domain $D(T)$, $T > \frac{1}{4}$, is bounded by two curves $l_1 = l_{11} \cup l_{12}$ and $l_2 = l_{21} \cup l_{22}$ having a mutual endpoint $i \in l_{11} \cap l_{21}$. The curve l_{11} in the complex (u, v)-plane is parameterized by the equations*

$$u(T) = \frac{C_0^2(\varphi, T)(4T-1) + (1-\sin\varphi)^2}{2C_0(\varphi, T)\cos\varphi}, \quad v(T) = \frac{1-\sin\varphi}{C_0(\varphi, T)}, \quad \varphi_0(T) \le \varphi \le \frac{\pi}{2}.$$

The curve l_{12} is parameterized by the equations

$$u(T) = \frac{C_{00}^2(\varphi, T)(4T-1) + (1-\sin\varphi)^2}{2C_{00}(\varphi, T)\cos\varphi}, \quad v(T) = \frac{1-\sin\varphi}{C_{00}(\varphi, T)}, \quad \varphi_0(T) \le \varphi \le \frac{\pi}{2}.$$

The curve l_2 is symmetric to l_1 with respect to the imaginary axis.

The same approach was used to draw similar value regions for inverse functions $f^{-1} : \mathbb{H} \to \mathbb{H} \setminus K(T)$.

We observe many value region problems solved in [4-6]. For instance, the authors determine value sets $\{f(z_0)\}$, $z_0 \in \mathbb{D}$, and $\{f^{-1}(w_0)\}$ for the class

$$\mathcal{I} = \{f \in \mathcal{H} : f(-\overline{z}) = -\overline{f(z)}, z \in \mathbb{H}\},$$

where $\mathcal{H}$ is the class of univalent self-mappings of $\mathbb{H}$ with the hydrodynamic normalization. Going to self-maps of $\mathbb{D}$, they prove a result which is equivalent to the classical solution by Goryainov and Gutlyanski [1] in the class $S(M)$, $M > 1$, $S(\infty) = S$, of univalent functions f in $\mathbb{D}$, $f(0) = 0$, $f'(0) = 1$, and $|f(z)| < M$ in $\mathbb{D}$. The same is done for typically real functions in $\mathbb{D}$ and some other classes.

Remind a theorem by Fedorov [3] which gives a value region $\{f(z_0)\}$, $z_0 \in \mathbb{D}$, over the class S_R of functions $f \in S$ with real values $f^{(n)}(0)$, $n \ge 2$. Obviously, an answer is easy if z_0 is real. From the other hand, it is strongly nontrivial when $\operatorname{Im} z_0 \ne 0$. Fedorov's result is extended in [8] to the class $S_R(M) = S_R \cap S(M)$. Usually, a subclass is organized more properly than a whole class of functions. However, it is not the case when we pass from S to $S(M)$ or from S_R to $S_R(M)$. Fedorov completely solved the problem by simultaneously considering two moduli problems for pairs of homotopic classes of curves. In $S_R(M)$, the problem is formulated as a reachable set problem for the Hamilton system of controllable differential equations in the frames of the Loewner theory. A family of Cauchy problems is substituted for the family of boundary value problems. The free parameter in the initial data serves as a parameter for the boundary curve of the value region. We do not write down a theorem proved in [8] since it requires too large volume.

Finally, let us concern with a class of holomorphic injective self-maps $f : \mathbb{D} \to \mathbb{D}$ having boundary fixed points, the class actively investigated during last decades by Goryainov, among others. For the dynamics of a self-map $f : \mathbb{D} \to \mathbb{D}$, a crucial role is played by the points $\sigma \in \partial\mathbb{D}$ for which $f(\sigma) := \angle \lim_{z\to\sigma} f(z) = \sigma$ and the angular derivative $f'(\sigma)$ is finite. Such points σ are called boundary regular fixed points. In particular, a classical result due to Wolff and Denjoy asserts that if $f \in \operatorname{Hol}(\mathbb{D}, \mathbb{D})$ has no fixed points in $\mathbb{D}$, then it possesses the so-called Denjoy-Wolff point, i.e., a unique boundary fixed point τ such that $f'(\tau) \le 1$.

Consider univalent self-maps $f : \mathbb{D} \to \mathbb{D}$ with a given boundary regular fixed point $\sigma \in \partial\mathbb{D}$ and the Denjoy-Wolff point $\tau \in \partial\mathbb{D} \setminus \{\sigma\}$. Using automorphisms of $\mathbb{D}$, we may suppose that $\tau = 1$ and $\sigma = -1$. We mean to determine a sharp value region of $f \mapsto f(z_0)$, $z_0 \in \mathbb{D}$, for all such self-maps of $\mathbb{D}$ with $f'(-1)$ fixed. Fix $z_0 \in \mathbb{D}$, $T > 0$ and let $\zeta_0 = x_1^0 + ix_2^0 := l(z_0)$, where

$$l : \mathbb{D} \to \mathbb{S}; \quad z \mapsto \log\frac{1+z}{1-z}$$

is a conformal map of $\mathbb{D}$ onto the strip $\mathbb{S} := \{\zeta\colon -\pi/2 < \operatorname{Im}\zeta < \pi/2\}$. Define

$$a_\pm(T) := e^{-T/2}\sin x_2^0 \pm (1-e^{-T/2}), \quad R(a,T) := \log\frac{1-a}{1-a_+(T)}\ \log\frac{1+a}{1+a_-(T)},$$

$$V(\zeta_0,T) := \left\{x_1+ix_2 \in \mathbb{S}\colon a_-(T) \le \sin x_2 \le a_+(T),\ \left|x_1 - x_1^0 - \tfrac{T}{2}\right| \le \sqrt{R(\sin x_2, T)}\right\}.$$

Theorem 3. *Let $f \in Hol(\mathbb{D},\mathbb{D}) \setminus \{id_{\mathbb{D}}\}$ and $T > 0$. Suppose that:*
(i) f is univalent in $\mathbb{D}$;
(ii) the Denjoy-Wolff point of f is $\tau = 1$;
(iii) $\sigma = -1$ is a boundary regular fixed point of f and $f'(-1) = e^T$.
Then

$$f(z_0) \in \mathcal{V}(z_0,T) := l^{-1}\big(V(l(z_0),T)\big) \setminus \{z_0\}$$

for any $z_0 \in \mathbb{D}$. This result is sharp, i.e., for any $w_0 \in \mathcal{V}(z_0,T)$ there exists $f \in Hol(\mathbb{D},\mathbb{D}) \setminus \{id_{\mathbb{D}}\}$, satisfying (i)–(iii) and such that $f(z_0) = w_0$.

Characterize functions f corresponding to boundary points of $\mathcal{V}(z_0,T)$. The role of the Koebe function $f_0(z) = z(1-z)^{-2}$in S is played by the Pick function $p_M(z) := f_0^{-1}(f_0(z)/M)$, $M > 1$.

Theorem 4. *For any $w_0 \in \partial V(z_0,T)\setminus\{z_0\}$, there exists a unique $f = f_{w_0}$ satisfying conditions (i)-(iii) in Theorem 3 and such that $f_{w_0}(z_0) = w_0$. If $w_0 = l^{-1}(\zeta_0 + T)$, then f_{w_0} is a hyperbolic automorphism of $\mathbb{D}$, namely, $f_{w_0}(z) = l^{-1}(l(z)+T)$. Otherwise, f_{w_0} is a conformal mapping of $\mathbb{D}$ onto $\mathbb{D}$ minus a slit along an analytic Jordan curve γ orthogonal to $\partial\mathbb{D}$, with $f'_{w_0}(1) = 1$. Moreover, $f_{w_0} = h_1 \circ p_M \circ h_2$ for some $h_1, h_2 \in Aut(\mathbb{D})$ and $M > 1$ if and only if $w_0 = l^{-1}(x_1^0 + T/2 + i\arcsin a_\pm(T))$.*

Note that z_0 is a boundary point of the value region $\mathcal{V}(z_0,T)$, but does not belong to $\mathcal{V}(z_0,T)$. This point z_0 would be included, and this would be the only modification of the value region, if we replace the equality $f'(-1) = e^T$ in condition (iii) of Theorem 3 by the inequality $f'(-1) \le e^T$ and remove the requirement $f \ne \mathrm{id}_{\mathbb{D}}$ assuming as a convention that $\mathrm{id}_{\mathbb{D}}$ satisfies (ii). Note also that, under the conditions of Theorem 3 modified in this way, $f(z_0) = z_0$ if and only if $f = \mathrm{id}_{\mathbb{D}}$.

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Universal Teichmüller space: non-trivial example of infinite-dimensional complex manifolds

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Since there is no well-developed theory of infinite-dimensional complex manifolds at the moment it is important to have different examples of such manifolds. One of non-trivial examples is given by the universal Teichmüller space. In our talk we present main complex geometric features of this manifold. The universal Teichmüller space $\mathcal{T}$ is the space of normalized quasisymmetric homeomorphisms of the unit circle S^1, i.e. orientation-preserving homeomorphisms of S^1, extending to quasiconformal maps of the unit disk Δ and fixing three points on S^1. It is a complex Banach manifold with the complex structure provided by Bers embedding of $\mathcal{T}$ into the complex Banach space of holomorphic quadratic differentials in a disk. The name of $\mathcal{T}$ is motivated by the fact that all classical Teichmüller spaces $T(G)$, associated with compact Riemann surfaces, are contained in $\mathcal{T}$ as complex subspaces. Another important subspace of $\mathcal{T}$ is given by the space $\mathcal{S}$ of normalized orientation-preserving diffeomorphisms of S^1. The space $\mathcal{S}$ is a Kähler Frechet manifold provided with the symplectic structure compatible with the complex structure of $\mathcal{S}$. There is an important Grassmann realization of $\mathcal{T}$ obtained by the embedding of $\mathcal{T}$ into the Grassmann manifold of a Hilbert space which coincides with the Sobolev space $V = H_0^{1/2}(S^1, \mathbb{R})$ of half-differentiable functions on the circle. This embedding realizes the group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of S^1 as a subgroup of symplectic group $\mathrm{Sp}(V)$. It also defines an embedding of $\mathcal{T}$ into the space of complex structures on V compatible with symplectic structure. The latter space may be considered as an infinite-dimensional Siegel disk.

The art of symmetrization: Ideas of I.P. Mityuk and modern developments

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The first symmetrization transformation was introduced by Jacob Steiner in 1838 in his attempt to find a geometric proof of the classical Isoperimetric Problem. It is amazing that for almost two centuries after its creation the method of symmetrization remains an incredibly powerful tool in many areas of mathematics and mathematical physics. Several ramifications and generalizations of this method were suggested by George Pólya, Gabor Szegö, Walter Hayman, Igor P. Mityuk, Al Baerstein II, and some other outstanding mathematicians.

In this talk, we focus on work and ideas of Prof. I.P. Mityuk who suggested several new approaches in the theory of symmetrization. It is important to mention that Prof. Mityuk was very generous to share his ideas with his students and, in fact, he created a school of mathematicians, based at the Kuban State University, who work in the area of symmetrization and its applications and produced many outstanding results in this area of mathematics. In particular, many interesting papers where symmetrization was used were published by V. Dubinin, V. Schlyk, B. Levitsky, Yu. Chernyh, M. Gavrilyuk, V. Tul'chii and some other former students of Prof. Mityuk. This author, being one of these students, also contributed to this area.

Three main themes having origins in Mityuk's works will be mentioned in this talk.

First, we will discuss symmetrization principle for multiply-connected domains, which was suggested by I.P. Mityk in 1960's and later found important applications to the study of different classes of analytic and meromorphic functions. In particular, we will discuss recent works of D. Betsakos, S. Pouliasis and Th. Ransford, which heavily depend on Mityuk's ideas, where the authors study analytic and geometric properties of rational and more general meromorphic functions.

Another very fruitful idea of I.P. Mityuk was to use conformal transformations to define symmetrization with respect to families of curves other then straight lines and circles. This idea leads to spiral symmetrization used by I.P. Mityuk and V. Schlyk and to symmetrizations with respect to trajectories of certain quadratic differentials used by V. Dubinin and this author. Several recent results obtained in this direction also will be mentioned in this talk.

The third topic influenced by the work of I.P. Mityuk, which will be discussed here, is the averaging and ordering transformations of symmetrization type. In this part, some results of V. Dubinin and results obtained by this author will be mentioned.

At the end of our talk we present some open questions on transformations of symmetrization type.

MSC 14R15.

Keller mappings and Jacobian conjecture[1]

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Let $f = (f_1, \dots, f_n)$ be a polynomial mapping $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ (or $\mathbb{C}^n \longrightarrow \mathbb{C}^n$), namely, $f_k(X)$ are polynomials, $X = (x_1, \dots, x_n) \in \mathbb{R}^n$.

The Jacobian Conjecture formulated in its modern form is: *if* $J_f(X) \equiv const \neq 0$, *then* f *is an injective mapping.*

Originally, Keller formulated the conjecture in 1939 for polynomial mappings with integer coefficients only if $n = 2$. General case study started later. There are many publications concerning this conjecture. The interest is mostly caused by its applications to differential equations. The conjecture has neither been proved (even for $n = 2$) nor rejected. It is included in the list of “Mathematical Problems for the Next Century” [1].

Various concepts of the conjecture are being developing during past decades. The most important one (at least for applications) is to find such polinomial mappings, for which The Jacobian Conjecture is true. Let us mention several known results.

1) The conjecture is proved for $n = 2$, if $(deg f_1, deg f_2) = 1$ [2].

2) The conjecture is tested for polynomial mappings with the highest degree $deg f$ of polinomials f_k not less than 100 in the case $n = 2$ [3].

3) The conjecture is proved for any n, if $deg f \leq 2$ [4].

4) The conjecture turns out to be true for $n \leq 4$ and $f = (f_1, \dots, f_n)$, where $f_k = x_k + H_k(X)$, $k = 1, \dots, n$, $H_k(X)$ are homogeneous polynomials of degree 3 (see [5]).

5) In 1998 E. Hubbers proved the conjecture for $n \leq 7$ and mappings $f = (f_1, \dots, f_n)$, where $f_k = x_k + (\sum_{j=1}^n a_{kj} x_j)^3$, $k = 1, \dots, n$.

The report contains new results by the author and co-authors in this field.

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Refinements of Schwarz Lemma and their applications to concave functions

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The idea of a multi-point Schwarz-Pick Lemma was first introduced by Beardon and Minda [BM]. It can be regarded as an extension of the Schur algorithm and enables us to obtain various Schwarz-type lemmas, see [CKS]. By using some of those results, we determine the coefficient bodies of the class B_p of analytic self-maps φ of the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ fixing a given point p with $0 < p < 1$. A meromorphic function f on $\mathbb{D}$ is called concave if f is univalent and if $\mathbb{C} \setminus f(\mathbb{D})$ is convex. Let Co_p be the class of concave functions f with a pole at $p \in (0, 1)$ normalized by $f(0) = 0$ and $f'(0) = 1$. In recent years, this class has been studied intensively by Avkhadiev, Bhowmik, Wirths and so on. It is known that for each $f \in Co_p$, there exists a $\varphi \in B_p$ such that

$$f'(z) = \frac{p^2}{(z-p)^2(1-pz)^2} \exp \int_0^z \frac{-2\varphi(t)}{1 - t\varphi(t)} dt, \quad z \in \mathbb{D}.$$

Conversely, for a given $\varphi \in B_p$, there exists a function $f \in Co_p$ satisfying the above formula.

By plugging this characterization with the information about the coefficient bodies of B_p, we investigate some coefficient problems for the class Co_p. This is based on the joint work with Rintaro Ohno (Tohoku University) [OS1, OS2].

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Quasiconformal analysis of two-indexed scale of spatial mappings and its applications [1]

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We define a family of mappings depending on two real parameters $n-1 \le q \le p < \infty$ and a weighted function θ. In the case $q = p = n$ and $\theta \equiv 1$ we obtain the well-known mappings with bounded distortion [1]. Mappings of two-indexed scale inherit many properties of the latter ones. They can be used for solving some problems of global analysis and some applied problems as well.

1. Results of papers [2–5] motivate a definition of the following class of mappings.

Definition 1. A mapping $f : \Omega \to \mathbb{R}^n$ is called the *mapping with bounded θ-weighted (q,p)-distrotion* (belongs to the class $\mathcal{ID}(\Omega; q, p; \theta, 1)$), $n-1 \le q \le p < \infty$, if:

1) f is continuous, open and discrete;
2) f belongs to Sobolev class $W^1_{n-1,\mathrm{loc}}(\Omega)$;
3) the Jacobian $J(x, f) \ge 0$ in Ω a. e.;
4) the mapping f has finite codistortion;
5) the function of the local θ-weighted (q,p)-distortion

$$\Omega \ni x \mapsto \mathcal{K}^{\theta,1}_{q,p}(x, f) = \begin{cases} \frac{\theta^{\frac{1}{q}}(x)|\operatorname{adj} Df(x)|}{J(x,f)^{\frac{n-1}{p}}}, & \text{if } J(x, f) \ne 0, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

belongs to $L_\varrho(\Omega)$ where ϱ is defined by the condition $\frac{1}{\varrho} = \frac{n-1}{q} - \frac{n-1}{p}$ ($\varrho = \infty$ at $q = p$).

We introduce the following notation $\mathcal{K}^{\theta,1}_{q,p}(f; \Omega) = \left\|\mathcal{K}^{\theta,1}_{q,p}(\cdot, f) \mid L_\varrho(\Omega)\right\|$. Here

1) $\theta : \mathbb{R}^n \to (0, \infty)$ is a measurable function (referred to be *weighted*);
2) for $n \times n$-matrix A, the symbol $\operatorname{adj} A$ denotes an adjoint matrix, defined by the condition $A \operatorname{adj} A = I \det A$ if the determinant of A is different of zero, and by the continuity in the topology of $\mathbb{R}^{n\times n}$ otherwise;
3) $J(x, f) = \det Df(x)$.

2. Following the paper [6, P. 265], for continuous, open, discrete and sense-preserving mapping $f : \Omega \to \mathbb{R}^n$ and a normal domain $D \Subset \Omega$, we define the Poleckiı̆ function $g_D : V \to \mathbb{R}^n$ on the image $V = f(D)$ as

$$V \ni y \mapsto g_D(y) = \Lambda \sum_{x \in f^{-1}(y)\cap D} i(x, f)x \quad \text{where } \Lambda \in (0, \infty) \text{ is a constant.} \tag{2}$$

New properties of the Poletskiı̆ function are formulated in the following statement.

Theorem 2 [7]. *Suppose that a mapping f belongs to $f \in \mathcal{ID}(\Omega; q, p; \theta, 1)$, $n-1 < q \le p < n + \frac{1}{n-2}$, and the weighted function $\omega(x) = \theta^{\frac{n-1}{q-(n-1)}}$ is locally integrable. Then*

1) *f is sense-preserving*;
2) *the Poleckiı̆ function g_D belongs to Sobolev class $W^1_s(V)$, $s = \frac{p}{p-(n-1)}$*;
3) *$f(B_f \cap D) \subset \{y \in V : Dg_D(y) = 0\}$ where B_f is a branch set.*

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By means of Theorem 2 we prove

Proposition 3. *Let $f : \Omega \to \mathbb{R}^n$ be a mapping of $\mathcal{ID}(\Omega; q, p; \theta, 1)$, $n-1 < q \leq p < n+\frac{1}{n-2}$, and the weighted function $\omega(x) = \theta^{-\frac{n-1}{q-(n-1)}}(x)$ is locally integrable. Then*

1) *f is differentiable in Ω a. e.;*
2) *$B_f \subset Z = \{x \in \Omega : J(x, f) = 0\}$ up to a set of measure zero, i. e. $|B_f \setminus Z| = 0$;*
3) *f is of finite distortion, namely $Df(x) = 0$ a. e. on a set where Jacobian vanishes.*

The following corollary concerns the weighted function $\theta \equiv 1$. In this case, q can take the value $n-1$. Moreover, a mapping $f \in \mathcal{ID}(\Omega; q, p; 1, 1)$, $n-1 \leq q \leq p \leq n$, has some additional properties.

Corollary 4. *Let $f : \Omega \to \mathbb{R}^n$ be a mapping of $\mathcal{ID}(\Omega; q, p; 1, 1)$, $n-1 \leq q \leq p \leq n$. Then*

1) *f has the Luzin property $\mathcal{N}^{-1}$: $|f^{-1}(E)| = 0$ if $|E| = 0$, $E \subset \Omega'$;*
2) *$J(x, f) > 0$ in Ω a.e.;*
3) *the branch set B_f has measure zero;*
4) *all mappings belonging to $\mathcal{ID}(\Omega; n, n; 1, 1)$ are mappings with bounded distortion.*

3. Theorem 2 serves as a basis for finding estimates for the norms of the push-forward functions, generalising the results of [5] on the mappings of $\mathcal{ID}(\Omega; q, p; \theta, 1)$. Using estimates for the norms of the push-forward functions, one can obtain estimates for a capacity similar to that of paper [5, Section 1.2]. Here we formulate only one of them.

Theorem 4. *Let $f \in \mathcal{ID}(\Omega; q, p; \theta, 1)$, $n-1 < q \leq p < \infty$, and a weighted function $\omega(x) = \theta^{-\frac{n-1}{q-(n-1)}}(x)$ be integrable. If $E = (A, C)$ is a condenser in Ω, $A \Subset \Omega$ and $C \subset A$ is a compact, then*

$$\bigl(\operatorname{cap}_s f(E)\bigr)^{1/s} \leq \mathcal{K}^{\theta,1}_{q,p}(f; \Omega)\bigl(\operatorname{cap}^{\omega}_r E\bigr)^{1/r} \tag{3}$$

where $s = \frac{p}{p-(n-1)}$ and $r = \frac{q}{q-(n-1)}$.

Recall

Definition 5. An ordered triple $E = (F_0, F_1; D)$ of non-empty sets, where D is an open set in $\mathbb{R}^n$, and F_1, F_0 are closed subsets of $\overline{D}$, is called the *condenser* in $D \subset \mathbb{R}^n$. The value

$$\operatorname{cap}^{\omega}_p(E) = \operatorname{cap}^{\omega}_p(F_0, F_1; D) = \inf \int\limits_D |\nabla v|^p(x)\omega(x)\, dx$$

where the infimum is taken over all functions $v \in C(D) \cap W^1_\infty(D) \cap L^1_p(D, \omega)$ such that $v \geq 1$ $(v \leq 0)$ in a neighbourhood of F_1 (F_0), is called *ω-weighted p-capacity of the condenser $E = (F_0, F_1; D)$*.

If U is an open set, C is a compact in U, then the condenser $E = (\partial U, C; U)$ will be denoted by $E = (U, C)$. If $\omega \equiv 1$ we consider a wider class of admissible functions: $v \in C(D) \cap L^1_p(D)$, for the definition of the capacity of the condenser.

By means of inequality (3) we prove many properties of mappings belonging to the class $\mathcal{ID}(\Omega; q, p; \theta, 1)$.

4. Here we formulate an assertion on removable singularities in the following sense.

Theorem 6. *Let*

1) *F be a closed set in Ω such the $\Omega \setminus F$ is locally connected at every point $x \in F$, $f \in \mathcal{ID}(\Omega \setminus F; q, p; \theta, 1)$, $n \leq q \leq p < \infty$ or $n-1 < q < n \leq p < \infty$,*
2) *the weighted function $\omega(x) = \theta^{-\frac{n-1}{q-(n-1)}}(x)$ is integrable on $\mathbb{R}^n$,*
3) *F has ω-weighted r-capacity zero in the space $W^1_r(\Omega; \omega)$ where $r = \frac{q}{q-(n-1)}$,*

4) $\operatorname{cap}\big(\mathbb{R}^n \setminus f(\Omega \setminus F); W_n^1(\mathbb{R}^n)\big) > 0$ *at* $p = n$.

Then the mapping f *can be extended to a continuous mapping* $\widetilde{f} : \Omega \to \overline{\mathbb{R}^n}$.

5. It is well-known that Riemannian manifolds can be classified by behavior of s-capacity at the "infinity". Recall, that a Riemannian manifold $\mathbb{M}$ is said to be *r-parabolic* (*θ-r-parabolic*) if $\operatorname{cap}_p(D, \mathbb{M}) = 0$ ($\operatorname{cap}_r^{\omega}(D, \mathbb{M}) = 0$) for any compact set $D \subset \mathbb{M}$ with nonempty interior. Otherwise, $\mathbb{M}$ is said to be *r-hyperbolic* (*ω-r-hyperbolic*).

Below, we formulate a statement on be behavior of parabolicity property under action of a mapping with bounded (θ)-weighted (q, p)-distortion.

Theorem 7. *Let* $f : \mathbb{M} \to \mathbb{N}$ *be a mapping with bounded* $(\theta, 1)$*-weighted* (p, q)*-distortion. If* $\mathbb{M}$ *is* ω*-*r*-parabolic then* $\mathbb{N}$ s*-parabolic, where* $r = \frac{q}{q-(n-1)}$, $s = \frac{p}{p-(n-1)}$, *and* $\omega(x) = \theta^{-\frac{n-1}{q-(n-1)}}(x)$ *is integrable.*

6. The collection of homeomorphisms $f : \Omega \to \Omega'$ of the $\mathcal{ID}(\Omega; q, n; 1, 1) \cap W_n^1(\Omega)$ where the domains Ω and Ω' have Lipschitz boundaries, can be considered as a class of admissible deformations in the non-linear elasticity theory. Note that the class of admissible deformations of the paper [8] is contained in the intersection $\mathcal{ID}(\Omega; q, n; 1, 1) \cap W_n^1(\Omega)$ for some $q > n - 1$, and class deformations of [9] coincides with $\mathcal{ID}(\Omega; n - 1, n; 1, 1) \cap W_n^1(\Omega)$. The new class of admissible deformations is a scale of families, depending on a continuous parameter $q \in [n - 1, n]$. These families are naturally ordered by inclusion: the class with bigger q is contained in the class with less q. For a given material, this hierarchy allows us to choose an appropriate class of deformations. In paper [10] we prove the existence of an extreme deformation of $\mathcal{ID}(\Omega; q, n; 1, 1) \cap W_n^1(\Omega)$ in a variational problem with some natural conditions on the growth of integrant.

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Metric properties of quasiconformal mappings

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This talk gives an overview of my recent research interests, connected with the theory of quasiconformal (qc) and quasiregular (qr) mappings in the Euclidean space $\mathbb{R}^n$, $n \geq 2$. When the important parameter K , the maximal dilatation of a mapping, tends to unity, we get these The talk will discuss the distortion theory of these mappings, i.e. how qc and qr maps transform distances between points. Some novel metrics are used in this research. The talk is based on joint work with several coauthors, mostly with my former students. In particular, the three latest coauthors are G. Wang, X. Zhang, and P. Hariri.

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Contributed Talks

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α–accessible domains[1]

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P. Liczberski and V. V. Starkov introduced in [1] the concept of an α–accessible domain. A domain $\Omega \subset \mathbb{R}^n$, $0 \in \Omega$, is called *α–accessible* (with respect to 0), $\alpha \in [0;1)$, if for any point $p \in \partial\Omega$ there exists a number $r = r(p) > 0$ such that the cone

$$K_+(p,\alpha,r) = \left\{ x \in \mathbb{R}^n : \ \|x\| \le r, \left(x - p, \frac{p}{\|p\|} \right) \ge \|x-p\| \cos\frac{\alpha\pi}{2} \right\} \subset \mathbb{R}^n \backslash \Omega.$$

α–accessible domains were shown in [1] to satisfy the cone condition and to be starlike. In the planar case ($n = 2$) such domains have been studied by J. Stankiewicz [2–3], D. A. Brannan, and W. E. Kirwan [4]. In [5], [6] K. F. Amozova and V. V. Starkov obtained the necessary and some sufficient conditions of α–accessibility in the nonsmooth case. In [7] K. F. Amozova and E. G. Ganenkova presented some properties of α–accessible domains and obtained the criteria of fully α–accessibility for p–harmonic functions in [8]. As a corollary, the criteria of fully α–accessibility for some classes of functions f were described in [8], including biharmonic ($n = 2$), harmonic ($n = 1$), analytic $\left(\text{the condition } \left|\arg\left(\frac{z f'(z)}{f(z)}\right)\right| \le \frac{\pi}{2}(1-\alpha), \text{ see [2–4]}\right)$, polyanalytic in $\mathbb{B} = \{z \in \mathbb{C} : |z| < 1\}$, and also the criteria of fully starlikeness ($\alpha = 0$) for p–harmonic, analytic $\left(\text{the well-known condition of starlikeness } \Re\left\{\frac{z f'(z)}{f(z)}\right\} \ge 0\right)$, and polyanalytic functions in $\mathbb{B}$.

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Three problems on extremal decomposition of the complex plane with free poles

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The report is devoted to extremal problems in geometric function theory of a complex variables associated with estimates of functionals defined on the systems of non-overlapping domains. Let $\mathbb{N}$, $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, $\overline{\mathbb{C}} = \mathbb{C} \bigcup \{\infty\}$ be a one point compactification and $\mathbb{R}^+ = (0, \infty)$. Let $r(B, a)$ be the inner radius of the domain $B \subset \overline{\mathbb{C}}$ with respect to a point $a \in B$. Consider an extremal problem which was formulated in paper [1] in the list of unsolved problems and then repeated in monograph [2].

Problem 1.[1] Consider the product

$$r^{\gamma}(B_0, 0) \prod_{k=1}^{n} r(B_k, a_k), \tag{1}$$

where B_0, B_1,..., B_n ($n \geq 2$) are pairwise disjoint domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$ and $0 < \gamma \leq n$. Show that it attains its maximum at a configuration of domains B_k and points a_k possessing rotational n-symmetry. In 1988 V. Dubinin solved this problem for $\gamma = 1$ and $n \geq 2$. In 1996 L. Kovalev got the solution to this problem for $n \geq 5$ and subclass points systems satisfying condition $0 < \alpha_k \leq 2/\sqrt{\gamma}$, $k = \overline{1, n}$, where $\alpha_k := \frac{1}{\pi} \arg \frac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1, n}$. In 2003 G. Kuz'mina studied this problem for $\gamma \in (0, 1]$. In 2008 A. Bakhtin [3] showed that the result of V. Dubinin holds for an arbitrary $\gamma \in \mathbb{R}^+$ but since some number $n_0(\gamma)$.

Problem 2.[1] Show that the maximum of the product

$$[r(B_0, 0)\, r(B_\infty, \infty)]^{\gamma} \prod_{k=1}^{n} r(B_k, a_k),$$

where $\gamma \in \mathbb{R}^+$, B_0, B_∞, $\{B_k\}_{k=1}^{n}$ are pairwise non-overlapping domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0, n}$, $\infty \in B_\infty \subset \overline{\mathbb{C}}$, is achieved for some configuration of the domains B_k, B_∞ and points a_k, ∞, $k = \overline{0, n}$, which are having n-fold symmetry. For $\gamma = \frac{1}{2}$ and $n \geq 2$ the problem 2 was solved in 1988 by V. Dubinin. In 2001 G. Kuz'mina showed that the Dubinin's estimate is correct when $\gamma \in \left(0, \frac{n^2}{8}\right]$, $n \geq 2$.

Problem 3.[1] Let $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$, $a_k \in B_k \subset \overline{\mathbb{C}}$, $k = \overline{0, n}$, where $B_0, \ldots, B_n$ are pairwise non-overlapping domains and $B_1, \ldots, B_n$ are symmetric domains with respect to the unit circle. Find the exact upper bound of the product (1). In 2000 L. Kovalev solved the problem 3 for $n \geq 2$ and $\gamma = 1$.

In the report a review of the latest results obtained in the above-mentioned problems will be presented.

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MSC 31C12

Solutions of the Poisson equation on model Riemannian manifolds

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We study the behavior of solutions of the Poisson equation on noncompact model Riemannian manifolds of a special type: $M = B \cup D_1 \cup D_2 \cup ... \cup D_p$, where B is precompactum with non-empty interior, and every D_i is isometric to the direct product $[r_0; +\infty) \times S^{n-1}$ (where $r_0 > 0$, S^{n-1} - sphere of dimension $n-1$) with metric

$$ds^2 = dr^2 + g_i^2(r)d\theta^2.$$

Here $g_i(r)$ is positive smooth functions on $[r_0; +\infty)$; $d\theta^2$ is a metric on S^{n-1}.

Losev A.G. in paper [1] was found exact conditions for the unique solvability of the Dirichlet problem for the Poisson equation of the following form:

$$\Delta u = f(r, \theta)$$

where f satisfies the following conditions:

- $f(r, \theta) \in C(D)$
- $\forall r \in [r_0; +\infty)\ f(r, \theta) \in C^{[\frac{3n}{2}]}(S^{n-1})$

However, in the classical statement of the problem on the right-hand side of the Poisson equation we apply weaker conditions. Appears the question, is the condition on f introduced by Losev A.G. necessary.
Define

$$K_i = \int\limits_{r_0}^{\infty} \frac{dz}{g_i(z)}$$

and

$$J_i = \int\limits_{r_0}^{\infty} g_i(t) \left(\int\limits_{t}^{\infty} \frac{dz}{g_i(z)} \right) dt.$$

In this paper for dimention $n = 2$ was already formulary the statement.

Theorem 1. *Let M such that for all i satisfied $K_i < \infty$, $J_i < \infty$. Then for any set of functions $\phi_i(\theta) \in C\,(S)$ and $f(\theta) \in C^{0,\alpha}(S^1)$, where $\frac{1}{2} < \alpha \leq 1$, there exists a unique $u(x)$ - solution of the Poisson equation such that*

$$\lim_{r\to\infty} u(r, \theta) = \phi_i(\theta).$$

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Quasiconformal extension of meromorphic functions with nonzero pole

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This talk is based on the articles [1] and [2]. It is well-known that the univalent functions defined in the unit disc that admit a quasiconformal extension to the extended complex plane play an important role in Teichmüller space theory. In this talk we consider meromorphic univalent functions f in the unit disc with a simple pole at $z = p \in (0, 1)$ which have a k-quasiconformal extension to the extended complex plane where $0 \leq k < 1$. We denote the class of such functions by $\Sigma_k(p)$. We first prove an area theorem for functions in this class. Next, we derive a sufficient condition for meromorphic functions in the unit disc with a simple pole at $z = p \in (0, 1)$ to belong to the class $\Sigma_k(p)$. Finally, we also provide a representation formula for functions in $\Sigma_k(p)$ and using this formula we derive an asymptotic estimate of the Laurent coefficients for the functions in this class.

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MSC 76F05.

Remarks on KO41 Theory of Turbulence[1]

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Let the turbulent flow be given by the velocity field $u(t,x)$, whose space-Fourier transform is denoted by $\hat{u}(t,\xi)$, where $\xi \in \mathbb{R}^3$ is interpreted as a frequency coordinate. The velocity field can be regarded as a random vector function. Omitting the symbol t from the notation, we introduce the energy distribution function with respect to the absolute value of the frequency:

$$\varphi(k) = \int_{|\xi|=k} |\hat{u}(\xi)|^2 dS\,.$$

The function φ is an analog of the well-known Planck function for the energy distribution in the blackbody radiation spectrum. The integral is taken over a sphere of radius $k \geqslant 0$.

Under the assumptions of an established (stationary) statistically homogeneous and isotropic turbulence under some additional assumptions about the structure of energy transformation (energy transfer from one part of the spectrum to another) Obukhov obtains that for the microcomponent of energy

$$E(p) = \int_p^{\infty} \varphi(k)dk$$

for $p \ll p_1 = \sqrt{\varkappa}\sqrt[4]{D_0/\nu^3}$ (see [3, 4]) the "two-thirds law" holds:

$$E(p) \approx const \cdot p^{-2/3}\,.$$

The "two-thirds" Kolmogorov's law [1, 2] can be represented by the following relation:

$$|u(\cdot+\ell) - u(\cdot)|^2_{L^2} \approx const \cdot \ell^{2/3}.$$

We discuss ways of obtaining these "laws", the degree of their equivalence, paying special attention to postulated assumptions. The connection with other turbulent "laws" is discussed. In addition, we pose and partially solve the problem of what can be proved in the theory of turbulence rigorously, that is, proceeding from the equations of motion?

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MSC 30E10.

On the Frobenius–Padé approximants,domains of their convergence and the supports of the equilibrium measures[1]

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Let s be a positive Borel measure on the interval $\Delta_\mu := [a;\, b] \subset \mathbb{R}$, let $p_k(x) = p_k(x;\, s)$, $k = 1, 2, \ldots$ be a system of corresponding orthonormal polynomials. Let f be a real-valued function in the class L^1_s, given by its Fourier expansion in system $\{p_k\}$:

$$f(x) \;=\; \sum_{k=0}^{\infty} c_k\, p_k(x), \quad c_k = c_k(f) = \int_a^b f\, p_k\, ds, \quad k = 0, 1, \ldots \tag{1}$$

For the orthogonal expansion (1) the *Frobenius–Padé approximant of index* $[L/M]$ is a rational function $\Phi_{L,M} = P/Q \in \mathcal{R}_{L,M}$ such that the first $(L + M + 1)$ Fourier coefficients of the linear form $(Q\, f - P)$ vanish:

$$c_k(Q\, f - P) = 0, \qquad k = 0, 1, \ldots, L + M.$$

We study the asymptotics of Frobenius–Padé approximants for the Markov-type functions f, which are the Cauchy transforms of some Borel measure σ on Δ_σ, where $\Delta_\sigma \cap \Delta_\mu = \varnothing$. The asymptotics are described in terms of the equilibrium problem of the vector potential with the Nikishin matrix of interaction. We set $c \in (0, 1/2]$. We denote by $\mathcal{M}_c$ the following class of Borel measures:

$$\mathcal{M}_c := \{(\mu, \sigma) :\ \mathrm{supp}(\nu) \subseteq \Delta_\nu,\ \nu \in \{\mu, \sigma\},\ |\mu| = 1,\ |\sigma| = c\}.$$

There exists a unique pair of measures $(\mu_c, \sigma_c) \in \mathcal{M}_c$ such that[2]

$$\begin{cases} 2V^{\sigma_c} - V^{\mu_c} \equiv \min_{\Delta_\sigma}(2V^{\sigma_c} - V^{\mu_c}) \ \text{ on } \ \mathrm{supp}(\sigma_c), \\ 2V^{\mu_c} - V^{\sigma_c} \equiv \min_{\Delta_\mu}(2V^{\mu_c} - V^{\sigma_c}) \ \text{ on } \ \mathrm{supp}(\mu_c). \end{cases}$$

We construct a three-sheeted Riemann surface and define a meromorhic function which correspons to the solution of this potential problem. Using the appropriate uniformization of the surface, we explicitly find the equilibrium measure in terms of an algebraic function.

In contrast to the diagonal case, where the convergence always takes place in the maximum possible domain, for all the non-diagonal ray sequences the divergence domains of approximants appear for certain arrangements of the intervals. Their occurrence is related to the pushing effect of the equilibrium measure, when the measure is zero on a part of the interval. We find explicitly the pushing point and the boundary of the convergence domain of the Frobenius–Padé approximants.

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[2]$V^\nu(z) = -\int \log|z - w| d\nu(w)$ is the logarithmic potential of a measure ν.

MSC 517.542

On Problem Fekete and Szegö

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We investigate an extremal problem of finding codomain of the functional $I(f) = c_3 - \gamma c_2^2$, where functions $f(z) = z + c_2 z^2 + c_3 z^3 + ...$ belong to class S of analytic and univalent functions in the unit disk. Szegö G., Fekete M. solved this problem [1] for $\gamma = \dfrac{p-1}{2p}$, $p \in \mathbb{N}$, investigating coefficients of p-fold symmetric functions (invariant under rotation). For any real γ, $0 \leqslant \gamma < 1$ using the Lowner method it is obtained [2] that the codomain of the functional is the disk of radius $R = 1 + 2e^{\frac{-2\gamma}{1-\gamma}}$ centered at the origin. Boundary functions were not found.

I.A. Alexandrov put the question of solving this problem with help of the variational method [3] and the problem of finding boundary functions. Using this method it is obtained for the boundary function of the functional $g = \mathrm{Re} I(f)$ the functional-differentiable equation

$$\frac{qf(z)+1}{f^4(z)} f'^2(z) = \frac{1 + qz + bz^2 + \overline{q}z^3 + z^4}{z^4},$$

where $q = (1-\gamma)2c_2$, $b = \dfrac{1}{3}\mathrm{Re} I(f)$.

The equation is analytically investigated. One of possible boundary functions is the function that maps the unit disk onto the plane slit along two analytical arcs connected at an angle $\alpha = \dfrac{2}{3}$. The slit takes beginning at infinity. For approximation of the functional's codomain it is used a family of functions $g = g(z, s, p, \psi, r)$, $g \in S$. The function g, $g(z) = \chi\left(w\big(h\left(\xi(z)\right)\big)\right)$, maps the unit disk onto the plane slit along a ray and a circular arc connected at angle $\alpha = \dfrac{2}{3}$, where $\chi(w) = \dfrac{1}{w'\big(h(\xi(0))\big)}\Big(w(h) - w\big(h(\xi(0))\big)\Big)$, $w(h) = \dfrac{h}{h-a}$, $h(\xi) = (1-\xi)^{\alpha}(1-\xi e^{ip})^{2-\alpha}\xi^{-1}$, $\xi(z) = e^{is}\dfrac{z - re^{i\psi}}{1 - re^{-i\psi}z}$, $|r| < 1$, $\psi, s, p \in \mathbb{R}$, $a = h\big(\xi(z_0)\big)$, z_0 – preimage of infinity under function g.

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Density of simple partial fractions and their generalizations in function spaces [1]

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Approximation properties of the simple partial fractions (logarithmic derivatives of polynomials)

$$r(z) = \sum_{k=1}^{n} \frac{1}{z - a_k}, \qquad a_k \in \mathbb{C},$$

had been studied intensively since 2001 [1].

The aim of the talk is to present several results concerning the problem of density of the sets $SF(E)$ of simple partial fractions with poles in a prescribed subset E of the complex plane in various function spaces.

We also consider the problem of density for various generalizations of simple partial fractions: the differences $SF(E^+) - SF(E^-)$ (logarithmic derivatives of rational functions with constraints on poles and zeros); the so-called λ-sums

$$S(f, E) = \left\{ \sum_{k=1}^{n} \lambda_k f(\lambda_k z) : \lambda_k \in E, n \in \mathbb{N} \right\},$$

where f is a fixed function analytic in some neighborhood of zero and $E \subset \mathbb{C}$; the sums

$$\sum_{k=1}^{N} f(t + a_k), \qquad a_k \in E, \quad N = 1, 2, \ldots,$$

of shifts of a single analytic function f, $E \subset \mathbb{C}$.

All results lie within the theory of density of semigroups in Banach spaces, which is developed by the author since 2014 [2].

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MSC 35N99
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Approximation of stationary solutions of the Navier-Stokes equations system with the potential of velocity and specific state equation for compressible continuum

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The class of solutions of the Navier–Stokes equations system for a compressible continuum under of state flow condition, potentiality of the velocity field, for a specific connection between density and pressure is considered. Solutions of this type can have a spatial periodicity, that is, displayed on the torus. With such a motion of the continuum, only volume viscosity appears in the flow, and the kinematic viscosity coefficient is assumed to be constant. The paper describes a set of such solutions (as class of functions), the properties of domain, and method of approximation by integrals along of specific curves.

It should be noted that an analytical study of the structure of this type of solutions that were considered, can provide information allowing to identify the class of solutions of Navier–Stokes system a more general form (with fewer assumptions). From a practical point of view, such solutions can be used for constructing of perturbation solutions of the Navier–Stokes equations of other types (exact solutions - analytical, asymptotic or generalized) and also for studying the properties of currents, where the extensional viscosity play the most significant role (relaxation – phase-unstable continua).

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Shur's algorithm for formal power series [1]

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Shur's algorithm for Shur's function f (i.e. $f \in H(\mathbb{D})$ and $|f(z)| \leq 1$) is based on the (finite or infinite) chain of equalities

$$f(z) = f(0) + \cfrac{(1-|f(0)|^2)z}{\overline{f(0)}z + \cfrac{1}{f_1(z)}}\,, \; f_1(z) = f_1(0) + \cfrac{(1-|f_1(0)|^2)z}{\overline{f_1(0)}z + \cfrac{1}{f_2(z)}}\,, \; f_2(z) = \dots . \quad (2)$$

It is easy to see that all functions $f_1, f_2, \dots$ are Shur's functions. The chain of equalities (1) is missing, if $|f(0)| = 1$ (in this case $f(z) \equiv f(0)$), and ends on the N-th step, if $|f_n(0)| < 1$ for all $n < N$ and $|f_N(0)| = 1$ (and therefore $f_N(z) \equiv f_N(0)$). Put $\gamma_0 := f(0)$, $\gamma_n := f_n(0)$, $n = 1, 2, \dots$, and for brevity rewrite the chain of equalities (1) in the following form

$$f(z) = [\gamma_0](z) \downarrow_{f_1(z)} = [\gamma_0, \gamma_1](z) \downarrow_{f_2(z)} = \dots = [\gamma_0, \dots, \gamma_{N-1}](z) \downarrow_{f_N(z)} = \dots . \quad (2)$$

Based on the chain of equations (2), Schur proved the following theorem.

Shur' Theorem. *Let f be a Shur's function. Then one of the following three statements is executed.*

(a) $f(z) \equiv \gamma_0$, $|\gamma_0| = 1$.

(b) There is a unique finite sequence $\{\gamma_n\}_{n=0}^N$ of complex numbers $|\gamma_n| < 1$, $n = 0, \dots, N-1$, $|\gamma_N| = 1$ such that

$$f(z) = [\gamma_0, \dots, \gamma_{N-1}](z) \downarrow_{\gamma_N} .$$

(c) There is a unique infinite sequence $\{\gamma_n\}_{n=0}^\infty$ of complex numbers $|\gamma_n| < 1$, $n = 0, 1, \dots,$ such that

$$f(z) = \lim_{n \to \infty} [\gamma_0, \dots, \gamma_{N-1}](z) \downarrow_{\gamma_N}$$

uniformly on compact sets lying in $\mathbb{D}$.

In the report we will enter into consideration two-point Hankel determinants Δ_n and Δ_n^* of Shur's function with which the parameters $\gamma_1, \gamma_2, \dots$ can be found without calculating functions $f_1, f_2, \dots$. Namely, in the assumption $f(0) \neq 0$ (that is not essential and is imposed only to simplify the formulation) the following equality is true

$$\gamma_n = (-1)^n f(0) \overline{\Delta}_n^* / \overline{\Delta}_n \,. \quad (3)$$

In addition, in terms of determinants Δ_n Shur's theorem specified as follows.

Case (a) *occurs if and only if* $\Delta_1 = 0$, *case* (b) – *if and only if* $\Delta_1 \neq 0, \dots, \Delta_N \neq 0$, $\Delta_{N+1} = 0$, *case* (c) – *if and only if* $\Delta_n \neq 0$ *for all* $n \in \mathbb{N}$.

It is easy to see that the Schur's algorithm can be applied not only to the Shur's function, but also to formal power series. In the report we will show that the formula (3) remains valid in this case. Besides we will discussed the corresponding analogue of Shur's theorem.

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Cohomological Gunning bundle for tori with punctures [1]

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Let F_0 be fixed compact Riemann surface of genus $g = 1$, with marked $\{a_1, b_1\}$, and distinct points $P_1, \ldots, P_m \in F_0$. Denote by $F'_\mu = F_\mu \backslash \{P_1, \ldots, P_m\}$ — surface of type $(1, m)$, $m \geq 2$, which have complex analytic structure by Beltrami differential $\mu(z) d\bar{z}/dz$ (on F_0). Let Γ'_μ — quasifuchsian group of first genus, which uniformizised F'_μ, with algebraic representation

$$\langle A_1, B_1, \gamma_1, \ldots, \gamma_m : [A_1, B_1]\gamma_1 \ldots \gamma_m = I \rangle,$$

where I — identical mapping. Denote by $\mathbb{T}_{1,m}(F'_0)$ Teichmueller space type $(1, m)$. Let $Z^1(\Gamma', \rho)$, for $\rho \in Hom(\Gamma', \mathbf{C}^*)$, set all mappings $\phi : \Gamma' \longrightarrow \mathbf{C}$ such that

$$\phi(ST) = \phi(S) + \rho(S)\phi(T), S, T \in \Gamma'.$$

Every element $\phi \in Z^1(\Gamma', \rho)$ uniquely be defined ordered set complex numbers

$$\phi(A_1), \phi(B_1), \phi(\gamma_1), \ldots, \phi(\gamma_m).$$

Space $B^1(\Gamma', \rho)$ generated element $\sigma(T) = 1 - \rho(T), T \in \Gamma'$.

Lemma 1. *For every $\phi \in Z^1(\Gamma', \rho)$ valid*

$$\sigma(B_1)\phi(A_1) - \sigma(A_1)\phi(B_1) + \phi(\gamma_1) + \sum_{j=1}^{m-1} \rho(\gamma_1 \ldots \gamma_j)\phi(\gamma_{j+1}) = 0.$$

Lemma 2. *Holomorphic principal $Hom(\Gamma', \mathbf{C}^*)$-bundle*

$E = \bigcup_{[\mu]} Hom(\Gamma'_\mu, \mathbf{C}^*)$ analytic equivalently trivial bundle $\mathbb{T}_{1,m}(F'_0) \times Hom(\Gamma', \mathbf{C}^*)$ over base $\mathbb{T}_{1,m}(F'_0)$.

Theorem. *Cohomological Gunning bundle*

$$G' = \bigsqcup_{([\mu], \rho)} Z^1(\Gamma'_\mu, \rho) \big/ B^1(\Gamma'_\mu, \rho)$$

over $\mathbb{T}_{1,m}(F'_0) \times (Hom(\Gamma', \mathbf{C}^*) \backslash \{1\})$ are holomorphic vector bundle of rang m for $m \geq 2$.

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About two nonlinear equations of 3 and 5 orders

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In articles [1], [2] consider a nonlinearly differential equations of third order

$$u_{xxx}u_y^3 - 3u_{xxy}u_y^2u_x + 3u_{xyy}u_yu_x^2 - u_{yyy}u_x^3 = 0 \tag{1}$$

$$u_{xxx}u_y^3 - 3u_{xxy}u_y^2u_x + 3u_{xyy}u_yu_x^2 - u_{yyy}u_x^3 +$$
$$+3\beta\sqrt{\left(u_x^2 + u_y^2\right)}\left(u_{xx}u_xu_y + u_{xy}\left(u_y^2 - u_x^2\right) - u_{yy}u_xu_y\right) = 0 \tag{2}$$

In these thesis we investigate solvability some boundary problems for these equations.

Lemma 1. *Let given domain $D \in \mathbb{R}^2$ with boundary $\Gamma = \left\{ax^2 + by^2 + c = 0\right\}$. Let function $u\left(ax^2 + by^2 + c\right) \in C^3\left(\overline{D}\right)$,*

$$u|_\Gamma = u_x|_\Gamma = u_y|_\Gamma = 0. \tag{3}$$

Then such function be non uniqueness solution boundary problem (3) for equation (1). For condition $a = b$ this function be non uniqueness solution boundary problem (3) for equation (2).

Remark 1. *For example, conditions of lemma valid for function $u\left(ax^2 + by^2 + c\right) = e^{-\left(x^2+y^2-\rho^2\right)} + \sqrt{2}\,\sin\left(\left(x^2 + y^2 - \rho^2\right) - \frac{\pi}{4}\right)$, boundary conditions (3) and domains D with boundary $\Gamma = \left\{x^2 + y^2 = \rho^2,\ \rho > 0\right\}$.*

Consider equations $e^{-t} + \sqrt{2}\,\sin\left(t - \frac{\pi}{4}\right) = 0, t = ax + by$ and two solutions $t_1 = 0$ and $t_2 = c,\ c \in \left(\pi + \frac{\pi}{4},\ \pi + \frac{\pi}{2}\right)$.

Lemma 2. *Let given domain D with boundary $\Gamma_1 = \{ax + by = 0,\ ax + by = c,\}$ and $\Gamma_2 = \{ax + by = 0,\ ax + by = \pi,\}$. Let function $u\left(ax + by\right) \in C^3\left(\overline{D}\right)$,*

$$u|_{\Gamma_1} = u_x|_{\Gamma_1} = u_y|_{\Gamma_1} = 0. \quad (4) \qquad\qquad u|_{\Gamma_2} = u_{xx}|_{\Gamma_2} = u_{yy}|_{\Gamma_2} = 0. \quad (5)$$

Then such function be non uniqueness solution boundary problem (4) (or (5)) for equation (1). For condition $a = b$ this function be non uniqueness solution boundary problem (4) (or (5)) for equation (2).

Remark 2. *For example, 1) conditions problems (4), (1) and (4), (2) satisfy function*

$$u\left(x + y\right) = e^{-(x+y)} + \sqrt{2}\,\sin\left(\left(x + y\right) - \frac{\pi}{4}\right).$$

2) conditions problems (5), (1) and (5), (2) satisfy function $u\left(x + y\right) = \sin(x + y)$.

Remark 3. *In article [3] consider Korteweg-de Vries equation of fifth order*

$$u_t - u_{xxxxx} + c_1\,\left(u^3\right)_x + c_2\,\left(\left(u_x\right)^2\right)_x + c_3\,\left(u\ u_{xx}\right)_x = 0.$$

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Quadrature formulas with variable nodes and Jackson-Nikolskii inequalities for rational functions[1]

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Sharp quadrature formulas of interpolation type for algebraic and trigonometric rational functions on circles and segments of the real axis are well-known due to works of Divruk, Rovba, Rusak, Osipenko, Min and others. Chebyshev-Markov fractions of the first and second kind are widely used to construct such formulas with *fixed* nodes.

Another approach, *the method of notches*, was recently proposed in [1]. Using this method, one can already construct sharp quadrature formulas with *variable* nodes depending on a certain parameter. Under an appropriate change of this parameter, every node continuously runs over the whole domain of integration. Due to this, in particular, one of the nodes may be chosen at a prescribed point of the domain of integration. This property is used in [1] to obtain (q, p)-inequalities of Jackson-Nikolskii type for rational functions and polynomials. Furthermore, the method of notches allows to construct examples of extremal rational functions and polynomials quite easily in some cases.

A certain shortcoming of quadrature formulas from [1] is that they take into account not the multiplicity of an individual pole but only a prescribed maximal multiplicity of poles. This fact significantly restricts the possibility to use theorems from [1] in applications. We prove a modified version of the quadrature formulas, which already takes into account the multiplicity of each pole separately. This allows to obtain (q, p)-inequalities more precise than in [1] and in other precedent papers. Let us formulate the main result in the case of the circle $\gamma := \{z : |z| = 1\}$.

Theorem 1. *Let $R(z)$ be a rational function of degree n whose poles do not belong to γ. Set*

$$\mathcal{R}(z) := R(z) \cdot \overline{R(1/\overline{z})}.$$

We denote by z_k, $k = 1, \dots, \nu$, all pairwise distinct poles of the rational function $\mathcal{R}$ which lie in the disc $|z| < 1$, and by n_k their multiplicities. Put

$$B(z) = \prod_{k=1}^{\nu} \left(\frac{z - z_k}{1 - z\overline{z_k}} \right)^{n_k}, \qquad \mu(z) = \sum_{k=1}^{\nu} \frac{n_k(1 - |z_k|^2)}{|z - z_k|^2}, \qquad \sum_{k=1}^{\nu} n_k = n.$$

For $m \in \mathbb{N}$ and $\varphi \in \mathbb{R}$ let the notches $\zeta_k(m, \varphi)$, $k = 1, \dots, mn + 1$, be the roots of the equation $\zeta B^m(\zeta) = e^{i\varphi}$. Then

$$\int_\gamma \mathcal{R}(\zeta)|d\zeta| = 2\pi \sum_{k=1}^{n+1} \frac{\mathcal{R}(\zeta_k)}{\mu(\zeta_k) + 1}, \qquad \zeta_k = \zeta_k(1, \varphi);$$

$$\|R\|_{L^{2m}(\gamma)}^{2m} := \int_\gamma |R(\zeta)|^{2m}|d\zeta| = 2\pi r \sum_{k=1}^{mn+1} \frac{|R(\zeta_k)|^{2m}}{m\mu(\zeta_k) + 1}, \qquad \zeta_k = \zeta_k(m, \varphi).$$

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The following corollary holds.

Theorem 2. *Under the assumptions of Theorem 1, the following* (q,p)*-inequality of Jackson-Nikolskii type is valid:*

$$\|R\|_{L^q(\gamma)} \le \left(\frac{m_p\,\|\mu\|_{L^\infty(\gamma)}+1}{2\pi r}\right)^{\frac{1}{p}-\frac{1}{q}} \|R\|_{L^p(\gamma)}, \qquad 0<p<q\le\infty, \tag{1}$$

where $m_p \in \mathbb{N}\cap[\frac{p}{2};1+\frac{p}{2})$. *Moreover, if all the poles of* R *do not belong to the annulus* $\left\{z : \delta < |z| < \delta^{-1}\right\}$, *where* $\delta\in(0,1)$, *then*

$$\|R\|_{L^q(\gamma)} \le \left(\frac{1}{2\pi}\right)^{\frac{1}{p}-\frac{1}{q}} \left(m_p\,n\cdot\frac{1+\delta}{1-\delta}+1\right)^{\frac{1}{p}-\frac{1}{q}} \|R\|_{L^p(\gamma)}, \qquad 0<p<q\le\infty. \tag{2}$$

The inequalities (1) *and* (2) *are sharp for* $(q,p)=(\infty,2)$.

The estimates (1) and (2) refine the corresponding ones in [1,2,3].

Finally let us note that Theorem 2 easily yields classical (q,p)-inequalities of Jackson-Nikolskii type for trigonometric polynomials T_n (see e.g. [4, Section 4.9.2]). For this it is sufficient to set $z=e^{it}$ and $\delta\to 0$ in the inequality (2). In this case the corresponding constant takes the form $((2m_p n+1)/(2\pi))^{\frac{1}{p}-\frac{1}{q}}$.

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Connected lemniscates and distortion theorems for polynomials and rational functions [1]

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In the present talk we discuss the impact of the connectivity of some lemniscates of a rational function f on the distortion of the mapping effected by f. We consider the distortion theorems for polynomials and rational functions [1]–[2], an inequality for the moduli of derivative of a complex polynomial at its zeros [3] and an inequality for the logarithmic energy of zeros and poles of a rational function [4]. All estimates obtained are sharp. The proofs of the theorems go by an application of a certain modification of the symmetrization method [5], for which the result of the symmetrization lies on the Riemann surface of the inverse function to a Chebyshev polynomial of the first kind.

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The equality of modulus and capacity of polycondenser in sub-Finsler space

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Let $\mathbb{G}$ be a Carnot group (see [1]). Polycondenser is a collection of sets $(\mathcal{F}, G) = \{(F_{01}, F_{11}), \ldots, (F_{0m}, F_{1m}), G\}$ where $G \subset \mathbb{G}$ is an open set and $F_{0k} \cap F_{1k} = \emptyset$ for all $k = 1, 2, \ldots, m$ (see [2]). Let $F(x, \xi)$, $H(x, \omega)$, $d\sigma$ be the same as in [1].

Let Γ be a family of oriented horizontal curves in G. The modulus of this family is $M_{p,F}(\Gamma) = \inf \int\limits_G \rho^p \, d\sigma$, where infimum is taken over all nonnnegative Borel functions ρ such that $\int\limits_\gamma \rho F(x, dx) \geq 1$ for all $\gamma \in \Gamma$. Modulus of polycondenser $M_{p,F}(\mathcal{F}, G)$ is a modulus of horizontal curves family joining F_{0k} and F_{1k} in G for all $k = 1, 2, \ldots, m$.

Capacity of polycondenser is $C_{p,F}(\mathcal{F}, G) = \inf \int\limits_G \max\limits_k H(x, \nabla u_k(x)) \, d\sigma$, where infimum is taken over all local Lipschitz functions u_k in G such that $u_k = j$ on a neighbourhood of F_{jk}, $j = 0, 1$, $k = 1, 2, \ldots, m$.

It was proved the equality of modulus and capacity of polycondenser:

Theorem.

$$M_{p,F}(\mathcal{F}, G) = C_{p,F}(\mathcal{F}, G).$$

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Classification theory for Klein surfaces

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In this article some classes of Klein surfaces are introduced taking into account the existence of analytic or harmonic functions. For instance, we shall consider the classes $O_K\left(HP\right), O_K\left(HB\right)$ and $O_K\left(HD\right)$ and establish parallel relations among them to the classical case of Riemann surfaces. It is also checked that on Riemann surfaces carrying an antianalytic involution, the theory of principal functions can be developed in a symmetric way so that we can apply these techniques to function theory on Klein surfaces.

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MSC 31C12

Massive sets and Liouville type theorems

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Let M be a smooth connected noncompact Riemannian manifold, Δ the Laplace - Beltrami operator on M, $\phi(\xi) \geq 0$, $0 \leq \xi < \infty$ weakly increasing, smooth functionis, not identically equal to zero. Considering the half linear equation on M,

$$Lu = \Delta u - u\phi(|u|) = 0, \tag{1}$$

we concern the following question: For what manifold M and function $\phi(\xi)$ does the equation have a unique solution u=0? If this is the case we say that Liouville's theorem is true for half linear equation.

The classical Liouville theorem says that any bounded harmonic function in $\mathbb{R}^n$ is constant. Recently there is more general approach to Liouville type theorems, namely, they are estimated dimensionality of various solutions spaces of linear equations elliptic type.

In particular, [1] proved an exact estimate dimensionality of spaces of bounded harmonic functions with finite Dirichlet integral $\int\limits_M |\nabla u|^2 dx < \infty$ on non-compact Riemannian manifolds in terms of massive sets.

Much latter [2] proved similar result for bounded solutions of the stationary equation Shrodinger.

The aim of this paper is to prove a similar result for bounded solutions of half linear equation.

We define LD-massive subsets of M and prove that the existance of bounded solutions (1) with finite Dirichlet integral

$$D(M, u) = \int\limits_M |\nabla u|^2 + 2\left(\int\limits_0^u t\phi(|t|)dt\right) dx, \tag{2}$$

is equivalent to the existance of LD-massive subset.

We now state the exact formulations.

A continuous function u defined on some open set $\Omega \subset M$ is called subsolution of (1) if for every subset $G \Subset \Omega$ and every function v

$$Lv = 0, u|_{\partial G} = v|_{\partial G}$$

implies $u \leq v$ in G.

An open proper subset $\Omega \subset M$ is called LD-massive if there is a non-trivial subsolution u of (1) such that $u = 0, x \in M \setminus \Omega, 0 \leq u \leq 1, D(M, u) < \infty$.

The main result of the paper is following statement.

Theorem 1. *The following statements are equivalent:*
i) On M exists non trivial bounded solution (1) with finite Dirichlet integral (2);
ii) On M exists LD-massive set.

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MSC 30A10.

Differential inequalities for polynomials[1]

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There are a lot of books and articles devoted to polynomial inequalities (see, [1-4], for example).

By $\mathbb{D}$ denote the unit disc.

In 1949, for fixed $n \in \mathbb{N}$ M. Marden introduced and investigated (see [2]) the following differential operator B, which associates with a polynomial $f(z)$ of degree at most n the polynomial

$$B[f](z) = \lambda_0 f(z) + \lambda_1 \frac{nz}{2} f'(z) + \lambda_2 \left(\frac{nz}{2}\right)^2 \frac{f''(z)}{2!},$$

here $\lambda_0, \lambda_1, \lambda_2$ are any constants such that

$$u(z) = \lambda_0 + C_n^1 \lambda_1 z + C_n^2 \lambda_2 z^2 \neq 0 \quad \text{in the half-plane} \quad \operatorname{Re} z > \frac{n}{4} \qquad (1).$$

Q. I. Rahman and G. Schmeisser showed that the operator B preserves the inequalities between polinomials. More precisely, they proved

Theorem A. [3, p. 538–540] *Let f, F be polynomials,* $\deg f \le \deg F = n$, F *have all zeroes in $\overline{\mathbb{D}}$, and $|f(z)| \le |F(z)|$ on $\partial\mathbb{D}$. Then for $z \in \mathbb{C} \setminus \mathbb{D}$*

$$|B[f](z)| \le |B[F](z)|, \qquad (2)$$

here the constants λ_k, defining the operator B, satisfy (1).

We have obtained a generalization of Theorem A. We have substantially expanded the possibility of choosing the parameters λ_k, for which inequality (2) holds.

Theorem 1. *Let f, F be polynomials from Theorem A, $R > 1$, L_R be the curve*

$$\left\{ \frac{n}{2}(1 + R\cos\varphi) \frac{Re^{i\varphi}}{(Re^{i\varphi})^2} : \varphi \in [0, 2\pi] \right\}.$$

If either a) zeroes z_1 and z_2 of the polynomial $u(z)$ belong to the half-plane

$$\operatorname{Re} z < \frac{n}{2} \frac{R}{1+R}$$

or b) the line, passing through z_1 and z_2, does not intersect L_R,
then for $|z| \ge R$ inequality (2) holds.

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MSC 30C70.

Maximum of the diameter of level lines in Jenkins class.

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Let Σ – class of univalent meromorphic functions $f(z) = z + a_0 + \frac{a_1}{z} + \dots$; in $U^* = \{z: \ |z| > 1\}$ $\Sigma_0 \subset \Sigma$, $f(z) \neq 0$. $\Sigma(r)$, $0 < r < 1$, subclass $f(z)$ from $\Sigma_0 : R(D,0) \ = \ r$, $D = \overline{C} \setminus \overline{f(U^*)}$, $R(D,0)$ conformal radius of simply connected domain D in the point 0. $\Sigma(\mu,\rho)$, $0 < \rho < 1$, $0 < \mu < -\frac{1}{2\pi}\log\rho$, subclass $f(z)$ from $\Sigma_0 : m\big(\overline{C} \setminus \overline{f(U^*)} \cup \overline{U_\rho}\big) = \mu$, $m(D)$ – modulus of double connected domain D, $U_\rho = \{z: \ |z| < \rho\}$. Class $\Sigma(r)$ is Jenkins class [1] studied in [1], [3], [4], [5]; $\Sigma(\mu,\rho)$ defined and studied in [5]. In class $\Sigma(\mu,\rho)$ obtained exact estimate of the diameter of level line by modulus method in combination with method of symmetrization. When $\rho \to 0$ Solynin A.Yu. [4] obtained respective result for class $\Sigma(r)$.

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Extremal bounds in module estimates

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The global conformality of a mapping provide a strong rigidity even on the plane. In higher dimensions this property holds, in view of the classical Liouville theorem, only for restrictions of the Möbius transformations. On the other hand, a natural extension of conformality is given by quasiconformal mappings, and the differentiability almost everywhere, Hölder continuty, etc. closely relate to weak conformality.

In the talk, we provide the sufficient conditions for homeomorphisms in $\mathbb{R}^n$ to be real differentiable at a point, Lipschitz and weakly Lipschitz continuous, etc. in the connection with the classical Teichmüller-Wittich-Beliskiĭ theorem. This involves a refined module technique. We present extremal bounds for distortions of the conformal moduli in both homeomorphic and nonhomeomorphic cases.

The analogues of Nehari theorem for harmonic functions [1]

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Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and h be a locally univalent function, analytic in $\mathbb{D}$. The Schwarzian derivative of h is defined as

$$S_h(z) = \left(\frac{h''(z)}{h'(z)}\right)' - \frac{1}{2}\left(\frac{h''(z)}{h'(z)}\right)^2.$$

An important role of S_h in geometric function theory follows from

Nehari Theorem [1]. *Let h be a locally univalent analytic function in $\mathbb{D}$ and $|S_h(z)| \le 2p(|z|)$ in $\mathbb{D}$, where the Nehari function $p(x)$ is positive, continuous, even on $(-1,1)$, and $(1-x^2)^2 p(x)$ is nonincreasing on $[0,1)$ and no non-trivial solution of the differential equation $u'' + pu = 0$ has more than one zero on $(-1,1)$. Then h is globally univalent in $\mathbb{D}$.*

The special case of Nehari theorem claims univalence of h if $|S_h(z)|(1-|z|^2)^2 \le 2$ in $\mathbb{D}$.

Every sense-preserving function $f(z)$, harmonic in the unit disk $\mathbb{D}$, has a form $f(z) = h(z) + \overline{g(z)}$, where h and g are analytic in $\mathbb{D}$ and the dilatation $\omega(z) = g'(z)/h'(z)$ is analytic, $|\omega(z)| < 1$ for all $z \in \mathbb{D}$. The order of f is defined [2] as

$$\alpha := \frac{1}{2}\sup_{z\in\mathbb{D}}\left|\frac{h''(z)}{h'(z)}(1-|z|^2) - 2\overline{z}\right|.$$

R. Hernández and M. J. Martín [3] proposed a definition of Schwarzian derivative for the sense-preserving harmonic mappings:

$$\mathbb{S}_f(z) = \left(\ln(|h'(z)|^2 - |g'(z)|^2)\right)_{zz} - \frac{1}{2}\left(\ln(|h'(z)|^2 - |g'(z)|^2)\right)_z^2.$$

The properties of $\mathbb{S}_f$ were intensively studied in the row of papers (cf., [3 – 5]).

During the talk some analogues of Nehari theorem for harmonic functions will be discussed. Particularly the following results will be presented:

Theorem 1 [5]. *Let a harmonic function f be sense-preserving in $\mathbb{D}$, $\alpha < \infty$ be an order of f and $|\mathbb{S}_f(z)| + \frac{2\alpha+19/2}{(1-|z|^2)^2} < 2p(|z|)$ for some Nehari function p and for all $z \in \mathbb{D}$. Then f is univalent in $\mathbb{D}$.*

Theorem 2. *Let a harmonic function f be sense-preserving, $|\omega(z)| \le k < 1$ in $\mathbb{D}$, $\alpha < \infty$ be an order of f and $|\mathbb{S}_f(z)|(1-|z|^2)^2 \le 2 - 8k(\alpha + 3 + 3k/(1+k^2))/(1+k^2)$ for all $z \in \mathbb{D}$. Then f is univalent in $\mathbb{D}$. Result of Nehari follows when $k \to 0$.*

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Approximation of derivatives based on Φ-triangulation

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The class of Φ-triangulations of a finite set of P points in $\mathbb{R}^n$ similar to Classical Delaunay triangulation is introduced. Such triangulations are constructed using the condition of empty intersection with the set P of the interior of every convex set from a given family of convex, bounded sets whose boundary contains vertices of the simplex of the triangulation. In this case, the classical Delaunay triangulation corresponds to the family of all balls in $\mathbb{R}^n$. We use of Φ-triangulations to obtain estimates of the error of approximation of the derivatives of C^2-smooth convex functions by piecewise linear functions.

Let $P \subset \mathbb{R}^n$ be a finite set of points $P = \{p_i\}$, $i = 1, ..., N$. For a function $f \in C^1(\mathbb{R}^n)$ we consider the problem of approximation of its derivatives by values $f(p_i)$. One of the methods for solving such a problem is a method based on the construction of a triangulation T of the set P. For a simplex $S \in T$ with vertexes $p_{i_0}, p_{i_1}, ..., p_{i_n} \in P$ there is the function $f_S(x) = \langle b, x\rangle + c$ such that

$$f(p_{i_k}) = f_S(p_{i_k}), \quad k = 0, ..., n.$$

We put

$$\delta_S(f) = \max_{x\in S} |\nabla f(x) - \nabla f_S(x)|. \tag{1}$$

We consider the function $f : \mathbb{R}^n \to \mathbb{R}$ of class $C^2(\mathbb{R}^n)$ such that

$$\left| \sum_{i,j=1}^{n} \frac{\partial^2 f(x)}{\partial x_i \partial x_j} \xi_i \xi_j \right| \le M|\xi|^2, \quad i, j = 1, ..., n \tag{2}$$

for all $\xi \in \mathbb{R}^n$ with some constant $M > 0$. For $r > 0$ and $y \in \mathbb{R}^n$ we define the following set

$$\Sigma_f(y, r) = \{x : f(x) + M|x - y|^2 < f(y) + \langle\nabla f(y), x - y\rangle + r\}. \tag{3}$$

Theorem. *Let $f(x) \in C^2(\mathbb{R}^n)$ be a function with (2). Then for every locally finite ε-net $A \subset \mathbb{R}^n$ and its Φ-trianguation T, constructed by family $\{\Sigma_f\}$ the following inequality holds for all simplexes $S \in T$*

$$\sup_{x\in S} |\nabla f(x) - \nabla f_S(x)| \le C(n)\frac{3M}{2}\sqrt{3}\varepsilon + 2MR(S),$$

where $R(S)$ is the circumradius of the simplex S, i.e., the radius of a sphere circumscribed around S.

MSC 30C62, 31A05.

On quasiconformal maps and semi-linear equations in the plane

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Assume that Ω is a domain in the complex plane $\mathbb{C}$ and $A(z)$ is symmetric 2×2 matrix function with measurable entries, $\det A = 1$ and such that $1/K\,|\xi|^2 \le \langle A(z)\xi, \xi\rangle \le K|\xi|^2$, $\xi \in \mathbb{R}^2$, $1 \le K < \infty$. In particular, for semi-linear elliptic equations of the form $\operatorname{div}(A(z)\nabla u(z)) = f(u(z))$ in Ω, where f is continuous and nonnegative, we prove Subharmonic factorization theorem. This theorem states that every weak solution u to the above equation can be expressed as $u = T \circ \omega$, where $\omega : \Omega \to G$ stands for a $K-$quasiconformal homeomorphism generated by the matrix function $A(z)$ and $T(w)$ is a weak subharmonic solution of the semi-linear equation $\triangle T(w) = J(w)f(T(w))$ in G. Here the weight $J(w)$ is the Jacobian of the inverse mapping ω^{-1}. Similar results hold for the corresponding nonlinear parabolic and hyperbolic equations. Some applications of these results in anisotropic media are given.

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MSC 30B50.

Representing systems of exponentials in locally convex subspaces of the normed space of analytic functions[1]

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Let D be a bounded convex domain in the complex plane containing the origin, and let $\mathcal{M} = (M_n)_{n=0}^{\infty}$ be a non-decreasing logarithmically convex sequence of positive numbers. By $H(D,\mathcal{M})$ we denote the Carleman class type space consisting of functions analytic in D with the norm

$$\|f\| = \sup_{n\geq 0}\sup_{z\in D}\frac{|f^{(n)}(z)|}{M_n} < \infty.$$

The system $\{e^{\lambda z}\}_{\lambda\in\mathbb{C}}$ is complete in the Banach spaces $H(D,\mathcal{M})$.

Let E be a normed subspace in $H(D)$. We call a largest locally convex space contained in E and invariant with respect to the operator of differentiation an invariant kernel $\mathcal{E}_p$. Let us $M_n = M_0$ for $-n \in \mathbb{N}$ and let us denote by $\mathcal{M}_k,\ k\in\mathbb{Z}$, the "shifted" sequences $(M_{n+k})_{n=0}^{\infty}$.

Theorem 1. *Let $\mathcal{M}$ be a non-decreasing logarithmically convex sequence of positive numbers satisfying the "non-quasi-analyticity" condition*

$$\sum_{k=0}^{\infty}\frac{M_k}{M_{k+1}} < \infty.$$

Then the invariant kernel $\mathcal{H}_p(D,\mathcal{M})$ of the space $H(D,\mathcal{M})$ coincides with $\bigcap_{-k\in\mathbb{N}} H(D,\mathcal{M}_k)$. If we consider the topology of the projective limit in this intersection, than the differentiation operator acts continuously in this space.

A discrete set $S = \{\mu_n\}_{n\in\mathbb{N}}$ is said to be a sparse set if for some $\delta > 0$ the disks $B_k(\delta) = B(\mu_k,\delta)$ are pairwise disjoint and the counting function of this set satisfies the ratio $\sup_t(\mu(2t)-\mu(t)) < \infty$.

Theorem 2. *Let D and $\mathcal{M}$ be the same as in theorem 1. Then for a positive function $m(t)$, $t>0$, increasing to infinity and satisfying the condition $m(2t) - m(t) \leq 1$, $t > 0$, there is a system of exponents $\Lambda = \{\lambda_k,\ k\in\mathbb{N}\}$, such that the system of exponentials $\{e^{z\lambda_k},\ k\in\mathbb{N}\}$ is a representing system in the invariant kernel $\mathcal{H}_p(D,\mathcal{M})$ of the space $H(D,\mathcal{M})$, that is, any function $f\in\mathcal{H}_p(D,\mathcal{M})$ can be represented as the sum of a series*

$$f(z) = \sum_{k=1}^{\infty} f_k e^{z\lambda_k}$$

converging in the topology of projective limit of the spaces $H(D,\mathcal{M}^{(k)})$, $-k\in\mathbb{N}$. If we remove from the system of exponents Λ a sparse subset $\{\eta_k,\ k\in\mathbb{N}\}$ with counting function $\eta(t)$ satisfying the condition

$$m(t) - \eta(t) \nearrow +\infty,$$

then the system of exponentials $\{e^{z\lambda_k},\ \lambda_k \in \widetilde{S} = S\setminus\{\eta_k,\ k\in\mathbb{N}\}\}$ remains to be a representing system. If $m(t)-\eta(t) \leq C < \infty$, $t\geq 0$, then the remaining exponential system may not be a representing system.

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Analogs of Korn's inequality on Heisenberg groups [1]

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Korn's inequality plays central role in the analysis of boundary value problems in linear elasticity. We establish analogs of Korn's inequality on Heisenberg groups $\mathbb{H}^n$ with the sub-Riemannian metric. For a mapping $w : U \subset \mathbb{H}^n \to \mathbb{R}^{2n}$ of the class $W_p^1(U; \mathbb{R}^{2n})$, we introduce the following generalizations of linear and deviator strains:

$$\mathcal{S}_1 w = \frac{D_h w + (D_h w)^t}{2}, \qquad \mathcal{S}_2 w = \frac{D_h w + (D_h w)^t}{2} - \frac{tr D_h w}{2n} I,$$

where $D_h w = (X_i w_j)_{ij=1..2n}$ is the horizontal differential. (Here $X_1, \dots, X_{2n}$ are basis horizontal orthonormal left-invariant vector fields.) We can generalize Korn's inequality in two different ways. First, we may estimate the horizontal differential via its symmetric part.

Theorem 1. *Let $U \subset \mathbb{H}^n$ be a John domain and $p > 1$. Then*

$$\|D_h w\|_p \leqslant C(\|w\|_p + \|\mathcal{S}_i w\|_p) \quad \textit{for } w \in W_p^1(U; \mathbb{R}^{2n}) \quad \textit{and}$$

$$\|D_h w\|_p \leqslant C\|\mathcal{S}_i w\|_p \quad \textit{for } w \in W_{p,O}^1(U; \mathbb{R}^{2n}), \quad \forall n \textit{ for } i = 1,\ n \neq 1 \textit{ for } i = 2.$$

Second approach generalizes the following fact: linear strain vanishes on the Lie algebra of the group of isometries. (Deviator strain vanishes on the Lie algebra of the group of conformal mappings.) Therefore, we want to construct differential operators $\mathcal{Q}_1$ and $\mathcal{Q}_2$ whose kernels coincide with the Lie algebra of the group of isometries and the Lie algebra of the group of conformal mappings on Heisenberg groups. To be more precise, kernels are horizontal coordinate functions of mappings from the Lie algebras. Define $\mathcal{Q}_i w = \begin{pmatrix} \mathcal{S}_i w \\ \mathcal{T} w \end{pmatrix}$, $i = 1, 2$, where

$$\mathcal{T} w = \begin{cases} \frac{1}{2}(D_h w + J D_h w J), & \text{for } n > 1,\ J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \\ \begin{pmatrix} X_2 X_1 w_2 - 2X_1 X_2 w_2 - X_1^2 w_1 \\ 2X_2 X_1 w_1 - X_1 X_2 w_1 + X_2^2 w_2 \end{pmatrix}, & \text{for } n = 1. \end{cases}$$

Theorem 2. *Let $U \subset \mathbb{H}^n$ be a John domain, $p > 1$, $w \in W_p^1(U; \mathbb{R}^{2n})$ for $n > 1$ and $w \in W_p^2(U; \mathbb{R}^2)$ for $n = 1$. Then $\|w - \Pi_i w\|_{1,p} \leqslant C\|\mathcal{Q}_i w\|_p$, where Π_i is a projection to the kernel of $\mathcal{Q}_i$, $i = 1, 2$.*

To prove Theorems 1 and 2 we use special integral representation formula from [1].

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Rational Bernstein- and Markov-type inequalities [1]

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In this talk we consider asymptotically sharp extensions of the classical Bernstein and Markov inequalities for rational functions on Jordan arcs and curves (see [1]). Also we discuss higher order inequalities for trigonometric polynomials (see [2]). The asymptotically sharp constants there can be expressed via the normal derivatives of certain Green's functions with poles at the poles of the rational functions in question. In the proofs key roles are played by Borwein-Erdélyi inequality on the unit circle, Gonchar-Grigorjan type estimate of the norm of holomorphic part of meromorphic functions, fast decreasing polynomials, and conformal mappings.

This presentation is based on a joint work with Béla Nagy and Vilmos Totik.

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On Metric Properties of Mappings of Sub-Lorentzian Structures [1]

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The aim of the talk is to describe in sub-Lorentzian geometry surface images of graph mappings constructed using intrinsically Lipschitz mappings of two-step Carnot groups. Sub-Lorentzian geometry can be considered as a sub-Riemannian version of well-known Minkowski geometry. The research in sub-Lorentzian geometry began only in recent years.

We consider $\varphi \in C^1_H(\Omega, \widetilde{\mathbb{G}})$, where $\Omega \subset \mathbb{G}$ is an open set, $\mathbb{G}$ ($\widetilde{\mathbb{G}}$) is a graded nilpotent group of the topological dimension N ($\widetilde{N}$) with Lie algebra $V = V_1 \oplus V_2$ ($\widetilde{V} = \widetilde{V}_1 \oplus \widetilde{V}_2$), and a unit element $\mathbf{0}$ ($\widetilde{\mathbf{0}}$). Moreover, $\mathbb{G}, \widetilde{\mathbb{G}} \subset \mathbb{U}$, where $\mathbb{U}$ is a two-step nilpotent graded group of the topological dimension $N + \widetilde{N}$, $\mathbb{G} \cap \widetilde{\mathbb{G}} = 0 = (\mathbf{0}, \widetilde{\mathbf{0}})$. The *graph mapping* $\varphi_\Gamma : \Omega \to \mathbb{U}$ assigns to each x the element $\varphi(x) \cdot x$, where "$\cdot$" is the group operation (defined by Baker–Campbell–Hausdorff formula).

The *sub-Lorentzian* distance on $\mathbb{U}$ is defined via following definition of the squared norm of a vector field (here $Y_1, \dots Y_{N+\widetilde{N}}$ are basis fields on $\mathbb{U}$):

$$\mathbf{d}_2^{SL^2}\Big(\sum_{j=1}^{N+\widetilde{N}} y_j Y_j\Big) = \max\Big\{ \sum_{j=1}^{\dim \widehat{V}_1^+} y_j^2 - \sum_{k=1}^{\dim \widehat{V}_1^-} y^2_{\dim \widehat{V}_1^+ + k},$$
$$\operatorname{sgn}\Big(\sum_{j=1}^{\dim V_2} y^2_{\dim \widehat{V}_1 + j} - \sum_{k=1}^{\dim \widetilde{V}_2} y^2_{\dim \widehat{V}_1 + \dim V_2 + k}\Big) \times$$
$$\times \Big| \sum_{j=1}^{\dim V_2} y^2_{\dim \widehat{V}_1 + j} - \sum_{k=1}^{\dim \widetilde{V}_2} y^2_{\dim \widehat{V}_1 + \dim V_2 + k} \Big|^{1/2} \Big\}.$$

The *intrinsic measure* ${}^{SL}\mathcal{H}^\nu_\Gamma$ is defined by applying Carathéodory construction to sub-Lorentzian balls in the intrinsic basis obtained by modifying the initial one.

Theorem 1. *The intrinsic sub-Lorentzian ${}^{SL}\mathcal{H}^\nu_\Gamma$-measure of the graph surface $\varphi_\Gamma(\Omega)$ can be calculated with the formula*

$$\int\limits_\Omega {}^{SL}\mathcal{J}(\varphi, v)\, d\mathcal{H}^\nu(v) = \int\limits_{\varphi_\Gamma(\Omega)} d\, {}^{SL}\mathcal{H}^\nu_\Gamma(y),$$

where sub-Lorentzian Jacobian ${}^{SL}\mathcal{J}(\varphi, v)$ is defined as follows:

$$\sqrt{\det\big(E_{\dim V_1} + (\widehat{D}\varphi)^+_{V_1,\widetilde{V}_1}(v)^*(\widehat{D}\varphi)^+_{V_1,\widetilde{V}_1}(v) - (\widehat{D}\varphi)^-_{V_1,\widetilde{V}_1}(v)^*(\widehat{D}\varphi)^-_{V_1,\widetilde{V}_1}(v)\big)} \times$$
$$\times \sqrt{\det\big(E_{\dim V_2} - (\widehat{D}\varphi)_{V_2,\widetilde{V}_2}(v)^*(\widehat{D}\varphi)_{V_2,\widetilde{V}_2}(v)\big)}.$$

Here $(\widehat{D}\varphi)^+_{V_1,\widetilde{V}_1}$ and $(\widehat{D}\varphi)^-_{V_1,\widetilde{V}_1}$ are parts of sub-Riemannian differential of φ corresponding to "positive" and "negative" directions in $\widetilde{\mathbb{G}}$.

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MSC 30E20.

Integration of forms over non-rectifiable paths with applications[1]

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Let Γ be simple closed non-rectifiable curve on the complex plane dividing it in two domains D^+ and D^-, $\infty \in D^-$. If function f is defined on Γ. We call a function $F(z)$ its integrator if (i) F is continuously differentiable in $\mathbb{C} \setminus \Gamma$, has compact support, and its first partial derivatives are integrable in $\mathbb{C}$; (ii) F has limit values from both sides $F^\pm$ on Γ satisfying relation

$$F^+(t) - F^-(t) = f(t), \quad t \in \Gamma.$$

Theorem 1. *If f satisfies the Hölder condition with exponent $\nu(t) > 0$ in a neighborhood of t for any $t \in \Gamma$, and*

$$\nu(t) > 1 - m(t), \quad t \in \Gamma,$$

where $m(t)$ is the Marcinkievicz exponent of the curve Γ at the point t (see [1], [2]), then the function f has an integrator.

Analogous result is valid for certain classes of functions f with singularities on Γ.

If both coefficients u, v of differential form $\omega = udz + vd\overline{z}$ have integrators U, V, then functional

$$C^1 \ni \phi \mapsto \int\limits_\Gamma \phi\omega := \iint\limits_{\mathbb{C}} \left(\frac{\partial V\phi}{\partial z} - \frac{\partial U\phi}{\partial \overline{z}} \right) dz\, d\overline{z}$$

is a generalization of the curvilinear integral $\int \phi\omega$ on the case where it is taken over non-rectifiable path. We apply this generalization for solving of the Riemann boundary value problem in domains with non-rectifiable boundaries for analytic and generalized analytic functions.

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MSC 30E25.

Riemann boundary value problem and some certain Beltrami equations[1]

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We obtain a Cauchy-type integral representation to solve Beltrami equations and apply it to the Riemann boundary value problem for that equations on non-rectifiable contours. There are some classical results [1, 2] for such problems for piecewise-smooth contours, but in non-rectifiable case we get some new results.

We solve the problem in the following statement **The R(0) statement**: The polynomial f of degree s, the curve Γ, and defined on it coefficients G, g are given. Find the solution ϕ of equation $\overline{\partial}\phi = \mu\partial\phi, \quad |\mu(z)| < 1$. vanishing at the infinity in $\mathbb{C} \setminus \Gamma$ such that it has limit values $\phi^{\pm}(t)$ from the left hand side and from the right hand side at any point $t \in \Gamma$, these limit values satisfy boundary value condition $\phi^{+}(t) = G(t)\phi^{-}(t) + g(t), \quad t \in \Gamma$, and $\phi \in H'(\Gamma, \lambda)$ with $\lambda(t) > \alpha_H(t) - 1$, $t \in \Gamma$, where α_H is the local Hausdorff dimension of Γ.

All following results was obtained in terms of so called Marcinkiewicz exponents, which are a kind of metric characteristics, invented by the author [3, 4].

As a main result we obtain **Theorem 1.** *Let f be a polynomial, coefficients G and g belong to $H(\Gamma, \nu)$, and G does not vanish on Γ. We denote the decrement of the argument of G on Γ as $2\pi\kappa$. If conditions $\nu(t) > 1 - \frac{1}{2}\mathfrak{ma}(\Gamma; t), \quad t \in \Gamma$. and $\alpha_H(t) - 1 < \min\left\{\nu(t), \frac{\mathfrak{ma}(\Gamma;t) - 2(1-\nu(t))}{\mathfrak{ma}(\Gamma;t)} \cdot \frac{1+\beta}{1-\beta}\right\}, \quad t \in \Gamma$, are satisfied, then there exist an exponent $\lambda(t)$ such that the set of solutions of Riemann problem $\phi^{+}(t) = G(t)\phi^{-}(t) + g(t), \quad t \in \Gamma$ for the Beltrami equation in the $R(0)$ statement has the following structure:*

- if $\kappa = 0$, then it consists of a unique solution;

- if $\kappa > 0$, then it is infinite, and formula of the general solution contains arbitrary polynomial of degree less than κ;

- if $\kappa < 0$, then it is either empty or contains a unique solution depending on fulfillment of $-\kappa$ linear solvability conditions.

All solutions and solvability conditions are representable in terms of Cauchy type integrals over a non-rectifiable curve Γ.

An analogous result is valid for the $R(m)$ statement.

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MSC 30C20.

Study of the surface of a generalized reduced module for multiply connected domain

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Classical task in the complex function theory concerns with the construction of the conformal mappings $F(w,a) = (w-a)f(w,a)$ from the regions D of the planar type and finite connectivity onto the unit disk with cuts along the arcs of prescribed form, namely, circular concentric arcs, radial slits, or their various disjoint combinations.

I.P. Mityuk [1] has proposed a way to define a generalized reduced modules connected with functions $F(w,a)$. The generalized reduced module,

$$M(w,D) = \frac{1}{2\pi}\ln|f(w,w)|, \tag{1}$$

of a multiply connected domain D at a point w will be called *Mityuk's function* with respect to the distinguished canonical domain.

Connection of the functions (1) with the exterior inverse boundary value problems goes back to F.D. Gakhov [2]. As it has appeared, the non-emptiness of the critical points set of the function $M(w,D)$ is equivalent to the solvability of the suitable exterior problem. The existence of critical points of Mityuk's function in the case of circular concentric slits has been proved by M.I. Kinder [3]. The case of circular and radial slits is studied in the present report: we construct the function $F(w,a)$ and establish the following

Theorem. *Let D be $(n+1)$-ly connected Jordan domain. Mityuk's function $M(w,D)$ with respect to the unit disk with circular and radial slits has at least one critical point when $n \neq 1$.*

We also discuss the classification problem for the critical points of Mityuk's function and the examples of an absence of such points.

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Laurent series in the classical Cartan's domains

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When studying the behavior of holomorphic functions near isolated points, the Laurent series is an important tool.

In a multidimensional space in certain cases, for example, in some areas of Hartogs (Hartogs-Laurent series) or in the product of discs, there are expansions of the Laurent character. Moreover, the domain of convergence such expansions are relatively complete Reinhart domains (see [1]).

The paper deals with the expansions into Laurent series of functions in classical domains of three types, where the points of the domain serve matrices are rectangular, symmetric and skew-symmetric, respectively ([2-4]).

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Koebe theorem for p-valent functions

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Let $M_p(\omega)$, $p \geq 2$, be the class of all holomorphic p-valent functions in the unit disk $U = \{z : |z| < 1\}$ with normalization $f(0) = 0$, $f(\omega) = \omega$, $0 < \omega < 1$. [1, Ch. 3.4]. Let $\mathcal{R}(f)$ denote the Riemann surface obtained as image of the unit disk $U = \{z : |z| < 1\}$ under the mapping f of class $M_p(\omega)$. For every function from the class $M_p(\omega)$, we obtain the maximal value $\rho(p, \omega)$, for which the Riemann surface $\mathcal{R}(f)$ contains an open k-valent disk, $k \leq p$, branching over the disk $|w| < \rho(p, \omega)$.

$$\rho(p, \omega) := \frac{\omega}{T_p\left[\frac{4\omega+(1+\omega)^2\cos(\pi/(2p))}{(1-\omega)^2}\right]},$$

where $T_p(z) = 2^{p-1}z^p + \ldots$ is the Chebyshev polynomial of the first type.

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Representations of the Second Kind of the Solutions to the First Order General Elliptic System in the Complex Form

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Denote by $D = \{z : |z| < 1\}$ the unit disk of the complex z-plane E, $z = x + iy$, $i^2 = -1$; $\Gamma = \partial D$ is the boundary of D; and $\overline{D} = D \cup \Gamma$.

We use the following standard functional spaces with the standard norms: $L_p(\overline{D})$, $W_p^k(\overline{D})$, $k = 0, 1, \ldots$, $p \geq 1$, $C_\alpha^k(\overline{D})$, $C_\alpha^k(\Gamma)$, $k = 0, 1, \ldots$, $0 < \alpha < 1$, $W_p^{k-\frac{1}{p}}(\Gamma)$.

We consider in D the first order general elliptic linear system in the complex form

$$\mathcal{D}w \equiv \partial_{\bar{z}} w + q_1(z)\partial_z w + q_2(z)\partial_{\bar{z}}\overline{w} + A(z)w + B(z)\overline{w} = R(z),$$

where $\partial_{\bar{z}} = 1/2(\partial/\partial x + i\partial/\partial y)$, $\partial_z = 1/2(\partial/\partial x - i\partial/\partial y)$, are derivatives in the sense of Sobolev; $q_1(z)$ and $q_2(z)$ are given complex functions;

$$|q_1(z)| + |q_2(z)| \leq q_0 = \text{const} < 1, \; z \in \overline{D};$$

$A(z)$, $B(z)$, $R(z) \in L_p(\overline{D})$, $p > 2$, also are given complex functions.

Denote by

$$Tf(z) = -\frac{1}{\pi} \iint\limits_D \frac{f(\zeta)}{\zeta - z} d\xi d\eta, \quad \zeta = \xi + i\eta;$$

$$\Omega(w) \equiv w(z) + T(q_1(z)\partial_z w + q_2(z)\partial_{\bar{z}}\overline{w} + A(z)w + B(z)\overline{w}).$$

Theorem 1. *If $q_1(z)$, $q_2(z) \in C(\overline{D})$, $A(z)$, $B(z) \in L_p(\overline{D})$, $p > 2$, then Ω is a (real) linear isomorphism of the Banach space $W_p^1(\overline{D})$.*

Theorem 2. *If $q_1(z)$, $q_2(z)$, $A(z)$, $B(z) \in C_\alpha^k(\overline{D})$, $k \geq 0$, $0 < \alpha < 1$, then Ω is a (real) linear isomorphism of the Banach space $C_\alpha^{k+1}(\overline{D})$.*

Theorem 3. *If $q_1(z)$, $q_2(z)$, $A(z)$, $B(z) \in W_p^k(\overline{D})$, $k \geq 1$, $p > 2$, then Ω is a (real) linear isomorphism of the Banach space $W_p^{k+1}(\overline{D})$.*

Theorem 4. *In the assumptions of theorem 2 for an arbitrary function $w(z) \in C_\alpha^{k+1}(\overline{D})$, $k \geq 0$, we have a priori estimate:*

$$\|w\|_{C_\alpha^{k+1}(\overline{D})} \leq \text{const} \left\{ \|\mathcal{D}w\|_{C_\alpha^k(\overline{D})} + \|w\|_{C_\alpha^{k+1}(\Gamma)} \right\},$$

where const *depends only on k, α and norms of the coefficients of the operator $\mathcal{D}$ in $C_\alpha^k(\overline{D})$.*

Theorem 5. *In the assumptions of theorems 1, 3 for an arbitrary function $w(z) \in W_p^k(\overline{D})$, $k \geq 1$ we have a priori estimate:*

$$\|w\|_{W_p^k(\overline{D})} \leq \text{const} \left\{ \|\mathcal{D}w\|_{W_p^{k-1}(\overline{D})} + \|w\|_{W_p^{k-\frac{1}{p}}(\Gamma)} \right\},$$

where const *depends only on k, p, and norms of the coefficients of the operator $\mathcal{D}$ in the corresponding spaces.*

An estimate of the distortion of the angles of the triangles of a triangulation under a diffeomorphism

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Let triangulation in the plane be given in the form of a set of triangles $\{T_k\}_{k=1}^N$. Suppose that there is a mapping $f : \mathbf{R}^2 \to \mathbf{R}^2$ given by the equations $u = u(x, y)$, $v = v(x, y)$. We assume that it is continuously differentiable and one-to-one. We introduce some notations. The Jacobian of the mapping f is denoted by $J = u_x v_y - u_y v_x$ and let $||f'||^2 = u_x^2 + u_y^2 + v_x^2 + v_y^2$. Assume that there is a constant M such that $\max\{|u_x|, |u_y|, |v_x|, |v_y|\} \le M$. We set

$$\lambda = \min_{1\le k\le N} \frac{\mathrm{area} T_k}{(\mathrm{diam} T_k)^2}.$$

Further, we assume that there exists a constant $C > 0$ such that for any points (x_1, y_1) and (x_2, y_2) there are inequalities

$$|u(x_2, y_2)-u(x_1, y_1)-u_x(x_1, y_1)(x_2-x_1)-u_y(x_1, y_1)(y_2-y_1)| \le C((x_2-x_1)^2+(y_2-y_1)^2)$$

and

$$|v(x_2, y_2)-v(x_1, y_1)-v_x(x_1, y_1)(x_2-x_1)-v_y(x_1, y_1)(y_2-y_1)| \le C((x_2-x_1)^2+(y_2-y_1)^2).$$

If we connect the images of all the vertices of the triangles T_k under the map f, we also obtain a set of triangles $\{T_k'\}_{k=1}^N$.

Theorem 1. *Let for some constant $J_0 > 0$ inequality $J \ge J_0$ is hold everywhere in $\mathbf{R}^2$. Let $d = \max_k \mathrm{diam} T_k$ and*

$$d < \frac{J_0 \lambda^2}{C(4M\lambda + 2Cd)}.$$

Then for any angle θ' of the triangle T_k' is there the inequality

$$\sin\theta' \ge \frac{J - \frac{C}{\lambda}(4M + d\frac{2C}{\lambda})d}{||f'||^2 + 4\frac{C}{\lambda}(2M + d\frac{C}{\lambda})d} \sin\theta,$$

where θ' is the angle corresponding to the angle θ in triangle T_k and the values J and $||f'||$ are computed in one of the vertices of the triangle T_k.

MSC 30C20.

The accessory parameters in the Schwartz equation[1]

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The research focuses on the problem of constructing holomorphic and univalent mappings of the complex half-plane onto circular polygons with the use of the Schwarz differential equation. The author specifies the generalisation of the P.P. Kufarev's method [1] to determine accessory parameters and preimages of polygon vertices. The Kufarev method for determining accessory parameters in the Christoffel-Schwarz integral [2], based on the Lewner differential equation, reduces the problem of finding preimages of vertices to the problem of ODE system integration. Some special cases of mappings of a half-plane onto circular polygons are obtained. The research investigates the connection between the accessory parameters of the map and the preimages of vertices. In some special cases, this connection is obtained.

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On approximation by differences of simple partial fractions[1]

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We study a problem of uniform approximation of constant functions $f(z) = c$ by differences of simple partial fractions [1], i.e., by logarithmic derivatives $R = r'/r$ of rational functions r, on compact subsets K of complex plane. Every difference is a finite sum of the form

$$\sum \frac{1}{z - z_j} - \sum \frac{1}{z - \tilde{z}_j},$$

therefore our problem can be treated in terms of electrostatics: we construct on K the constant electrostatic field due to electrons (at points $\tilde{z}_j$) and positrons (at points z_j), sf. [2].

For every constant $c \in \mathbb{C}$ we put $r = r_n(z) = Q(z)/Q(-z)$, where $Q(z) = Q_n(c; z) = L_n^{-2n-1}(cz)$ ($L_n^{-2n-1}(t)$ is a generalized Laguerre polynomial L_n^{α} with parameter $\alpha = -2n - 1$). It's easy to see, that the logarithmic derivative

$$R_n(c; z) = r_n'(z)/r_n(z)$$

is even. We prove, that

$$R_n(c; z) - c = (-1)^{n+1} \frac{z^{2n}}{Q(z)Q(-z)} \cdot \mu_n(c), \qquad \mu_n(x) := x^{2n+1} \left(\frac{n!}{(2n)!} \right)^2,$$

and for every $r > 0$ and $n + 1 \geq \max\{14;\ e|cr/2|^2\}$

$$\frac{2}{3} \cdot |z|^{2n} \mu_n(|c|) < |R_n(c; z) - c| < 2 \cdot |z|^{2n} \mu_n(|c|), \qquad |z| \leq r.$$

In partially, for every $n \geq 1$ and all zeroes z_k of polynomial $L_n^{-2n-1}(z)$ we have

$$|z_k| > 2\sqrt{(n + 1)/e}, \qquad 1 \leq k \leq n, \quad n \in \mathbb{N}.$$

Remark, that our problem related to the classical problem of rational approximations to e^{cz}.

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MSC 30F99, 41A21

Hermite–Padé approximants for meromorphic functions on a compact Riemann surface[1]

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In the talk we consider the problem of reconstruction of the values of a multivalued algebraic function with the help of Hermite-Padé polynomials of the first kind.

More precisely. Let $\mathfrak{R}$ be a compact Riemann surface and $\pi : \mathfrak{R} \to \widehat{\mathbb{C}}$ be an $(m+1)$-fold branched covering of the Riemann sphere $\widehat{\mathbb{C}}$, $m \geqslant 1$. Let f be a meromorphic function on $\mathfrak{R}$, and let the functions $1, f, f^2, \dots, f^m$ be independent over the field $\mathbb{C}(z)$ of rational functions. Let $\circ$ be an arbitrary point of $\mathfrak{R}$ that is not critical for the projection π. Without loss of generality we suppose that $\circ \in \pi^{-1}(\infty)$ and denote $\boldsymbol{\infty}^{(0)} := \circ$. In a neighbourhood of ∞ we set $f_\infty(z) := f(\pi_0^{-1}(z))$, where π_0 is the biholomorphic restriction of π to a neighbourhood of $\boldsymbol{\infty}^{(0)}$. For convenience we suppose that the germ $f_\infty(z)$ is holomorphic at ∞.

Let us define the *Hermite–Padé polynomials of the first kind* $Q_{n,0}, \dots, Q_{n,m}$ of order $n \in \mathbb{N}$ for the tuple of germs $[1, f_\infty, \dots, f_\infty^m]$ at the point $\infty \in \widehat{\mathbb{C}}$ in the following way: $\deg Q_{n,j} \leqslant n$, $j = 0, \dots, m$, at least one of $Q_{n,j} \not\equiv 0$, and the following asymptotic relation at ∞ holds true:

$$Q_{n,0}(z) + \sum_{j=1}^{m} f_\infty^j(z) Q_{n,j}(z) = O\left(\frac{1}{z^{m(n+1)}}\right) \quad \text{as } z \to \infty.$$

We find the limiting zero distribution of such Hermite–Padé polynomials $Q_{n,j}(z)$ and also the asymptotic behaviour of the quotients $\frac{Q_{n,j}(z)}{Q_{n,m}(z)}$, $j = 0, \dots, m-1$, which are called Hermite–Padé approximants of f_∞. With the help of such Hermite–Padé approximants we reconstruct the values of f on the first m sheets in so-called "Nuttall's partition" of the Riemann surface $\mathfrak{R}$ into $m+1$ sheets that was introduced in [1].

The talk is based on the joint work with E.M.Chirka, R.V.Palvelev, and S.P.Suetin [2]. The work was also discussed with N.G.Kruzhilin.

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MSC 30D10.

Singular points of sum of exponential monomials series[1]

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Let $\Lambda = \{\lambda_k, n_k\}_{k=1}^{\infty}$ be a sequence of different complex numbers, $|\lambda_{k+1}| \geq |\lambda_k|$, $k \geq 1$, $|\lambda_k| \to \infty$, $k \to \infty$. The series exponential monomials are considered

$$\sum_{k=1,n=0}^{\infty,n_k-1} d_{k,n} z^n e^{\lambda_k z}. \tag{1}$$

Let $\varphi_1, \varphi_2 \in [-2\pi, 2\pi)$, $\varphi_2 - \varphi_1 \in (0, 2\pi]$. We set

$$\Gamma(\varphi_1, \varphi_2) = \{\lambda = te^{i\varphi} : \varphi \in (\varphi_1, \varphi_2), t > 0\}.$$

Let $\Lambda(\varphi_1, \varphi_2)$ denote a sequence consisting of all pairs $\{\lambda_k, n_k\}$ such these $\lambda_k \in \Gamma(\varphi_1, \varphi_2)$.

Let K be a convex compact with a supporting function

$$H(\varphi, K) = \sup_{z \in K} \operatorname{Re}(ze^{-i\varphi}), \quad \varphi \in \mathbb{R}$$

and let $L(\varphi, K) = \{z : \operatorname{Re}(ze^{-i\varphi}) = H(\varphi, K)\} \cap \partial K$, $\varphi \in \mathbb{R}$. Let $\Phi(D)$ denote a set of angles φ such that $L(\varphi, K)$ be a segment.

The sequence $\Lambda = \{\lambda_k, n_k\}$ will be called a simple if $n_k = 1$ and $|\lambda_{k+1}| - |\lambda_k| \geq h$, $k \geq 1$ for some number $h > 0$.

Let

$$\underline{n}_0(\Lambda) = \lim_{\delta \to 0} \varliminf_{r \to \infty} \frac{n(r, \Lambda) - n((1-\delta)r, \Lambda)}{\delta r},$$

where $n(r, \Lambda)$ is the number of points λ_k taking into account their multiplicities n_k in the circle $B(0, r)$,

$$m(\Lambda) = \varlimsup_{k \to \infty} \frac{n_k}{|\lambda_k|}, \quad \sigma(\Lambda) = \varlimsup_{j \to \infty} \frac{\ln j}{|\xi_j|},$$

where $\{\xi_j\}$ is not decreasing sequence composed of points λ_k at that each λ_k occurring in it exactly n_k times.

Theorem 1. $\Lambda = \{\lambda_k, n_k\}$, $m(\Lambda) = \sigma(\Lambda) = 0$, *$D$ is a convex domain,* $-\varphi \in \Phi(D)$, $[z_1, z_2] \subseteq L(-\varphi, D)$. *Assume that there is* $\delta > 0$ *such that* $\Lambda(\varphi - \delta, \varphi + \delta)$ *is a simple sequence and*

$$2\pi \inf_{\alpha > 0} \underline{n}_0(\Lambda(\varphi - \alpha, \varphi + \alpha)) \geq |z_2 - z_1|.$$

Then there are $d_{k,n}$ *such that the series converges in* D *and its sum has no singular points on the interval* (z_1, z_2).

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Plurisubharmonic foundations of Teichmüller's theory

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It is well-known that plurisubharmonicity of the Kobayashi metric on a complex Banach manifold implies deep geometric and pluripotential features. This metric on Teichmüller spaces coincides with their intrinsic Teichmüller metric determined by extremal quasiconformal maps. Its plurisubharmonicity is proven in [1] only for the universal Teichmüller space.

In the talk, we establish that the Kobayashi-Teichmmüller metric $\tau_n(\psi_1, \psi_2)$ of Teichmmüller spaces $\mathbf{T}(0, n)$ for the punctured spaces (and simultaneously for some Riemann surfaces of positive genus) is logarithmically plurisubharmonic separately in each of its argument; hence, the pluricomplex Green function of $\mathbf{T}(0, n)$ equals $g_{\mathbf{T}}(\psi_1, \psi_2) = \log \tanh \tau_n(\psi_1, \psi_2) = \log k(\psi_1, \psi_2)$, where k is the norm of extremal Beltrami coefficient defining the Teichmmüller distance between the points ψ_1, ψ_2 in $\mathbf{T}(0, n)$. In addition, the differential (infinitesimal) Kobayashi metric $\mathcal{K}_{\mathbf{T}}(\psi, v)$ on the tangent bundle $\mathcal{T}\mathbf{T}(0, n)$ of $\mathbf{T}(0, n)$ is logarithmically plurisubharmonic in $\psi \in \mathbf{T}(0, n)$, equals the infinitesimal Finsler form $F_{\mathbf{T}}(\psi, v)$ of metric τ_n and has constant holomorphic sectional curvature -4.

This important fact has many interesting applications and opens the ways to create the pluripotential theory related to finite dimensional Teichmmüller spaces.

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The Morera boundary theorem in the space of rectangular matrices

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In this paper we consider the realization of a classical domain of the first type in the form of a Siegel domain of the second type defined in the space of rectangular matrices, and for this domain we prove a boundary analogue of the Morera theorem. The boundary analogues of Morera's theorem were considered in [1]-[3], and also in the monograph [4].They affirm the possibility of a holomorphic continuation of the function f from the boundary ∂D of the domain $D \subset \mathbb{C}^n$ under the condition that the integrals of f are equal to zero along the boundaries of analytic discs lying on ∂D

It is known that any bounded homogeneous domain (with respect to holomorphic automorphisms) in $\mathbb{C}^N$ has a realization in the form of a Siegel domain of the second type. In particular, the classical domain of the first type $\mathcal{R}_1$ is biholomorphically equivalent to a some Siegel domain of the second type, which is constructed with the help of the following construction.

Let U_1 be a square matrix of dimension $p \times p$, and U_2 is a matrix of dimension $p \times (q-p)$. In the space of pairs matrices (U_1, U_2) of complex dimension $N = pq$ we consider domain

$$\mathcal{D} = \{U = (U_1, U_2) \in \mathbb{C}[p \times q] : \mathrm{Im} U_1 - U_2 U_2^* > 0\},$$

where $\mathrm{Im} U_1 = \frac{1}{2i}(U_1 - U_1^*)$.

We denote the skeleton of this domain by

$$\mathcal{G} = \{U = (U_1, U_2) : \mathrm{Im} U_1 = U_2 U_2^*\}.$$

Consider the following embedding of the disk $\Delta = \{|t| < 1\}$ into the domain $\mathcal{D}$:

$$\{\Omega_t \in \mathbb{C}^{pq} : \Omega_t = \Phi(t\Phi^{-l}(\Lambda^0))t \in \Delta\}, \tag{1}$$

where $\Lambda^0 \in \mathcal{G}$. If Ψ is an arbitrary automorphism of the domain $\mathcal{D}$, then the set (1) under the action of this automorphism becomes an analytic disk with boundary on $\mathcal{G}$.

Theorem. *Let f be a continuous bounded function on $\mathcal{G}$. If for f the condition*

$$\int\limits_{\partial\Delta} f(\Psi(\Omega_t))\, dt = 0$$

for all Ψ automorphisms of the domain $\mathcal{D}$ and a fixed $\Lambda^0 \in \mathcal{G}$, then the function f extends holomorphically in $\mathcal{D}$ to the function $F \in \mathcal{H}^\infty(\mathcal{D})$ is continuous up to $\mathcal{G}$.

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On a certain characteristic of closed nondegenerate curves [1]

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This work provides a partial solution to one of the problems formulated by A. A. Agrachev in [1]. Namely, an upper bound for $\mu(n)$ is obtained, where $\mu(n)$ is the minimal possible multiplicity of a circle in $\mathbb{R}^n$ which admits a nondegenerate (that is, having a Frenet frame in each point) continuous (as a closed curve) deformation.

The first non-trivial case of this problem was studied by J. Milnor [2] and W. Fenchel [3], whose results imply that $\mu(3) = 2$.

Our results consist of: [i] a simple observation that suffices to conclude that for all positive integers m we have $\mu(2m) = 1$, [ii] more importantly, a method of constructing some odd-dimensional examples recursively (starting at $n = 3$, then proceeding from $n = 2m + 1$ to $n = 2m + 3$).

The first step of the aforementioned method produces the following upper bound:

Theorem 1. $\mu(5) \leq 7$.

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The variational principle in the theory of quasiconformal mappings

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Variational methods for quasiconformal mappings were first introduced by Belinsky P. [1] and have find various application by Biluta P. [2], Krushkal S. [3], Shiffer M. [4, 5], Shober G. [5, 6] and others. In our paper, variational problems for certain classes of quasiconformal mappings are considered.

We have established that if $q_j\colon \mathbb{C} \to \mathbb{R}$, $j = 1, 2$ are continuously differentiable functions such that $0 < q_1 < q_2$ and $q_1 + q_2 < 1$, then there exists a solution f of the Beltrami equation with two characteristics, which has logarithmic singularity at the selected point. In order to find a representation of the function f we solve the variational problem. This problem consist in constructing the quasiconformal mapping deliviring maximum value to the slightly varied Schiffer and Schober's functionals. With this mapping at hand we compose it with a solution of Beltrami equation of the second kind.

Following the method exposed at the article [5] of Schiffer and Schober we find (p, q) – analytic function with a logarithmic singularity, whose real part is a quasi-fundamental solution of the equation: $(pU_x)_x + (pU_y)_y + q_xU_y - q_yU_x = 0$.

Besides we introduce the class of K_α – quasiconformal mappings whose characteristics are unbounded at the given point z_0, coefficients of the corresponding systems tending to infinity as $|z - z_0|^{-\alpha}$, $z \to z_0$.

On this class of mappings we find an extremal one for the functional generalizing that of Schiffer and Schober.

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MSC 58E20, 31B15

P-harmonic radius and p-harmonic Green's mappings

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We consider the class of p-harmonic Green's mappings for which level surfaces $S_t(u_G) = \{x \in G : u_G(x, x_0) = t\}$ of p-harmonic Green's function $u_G(x, x_0)$ for the domain $G \subset E^n$ ([1]) are mapped on the level surfaces $S_t(u_{\widetilde{G}}) = \{y \in \widetilde{G} : u_{\widetilde{G}}(y, y_0) = t\}$ of p-harmonic Green's function $u_{\widetilde{G}}(y, y_0)$ for the domain $\widetilde{G} \subset E^n$, and the trajectory of the field $\nabla u_G(x, x_0)$, which enters the pole x_0, corresponds to the trajectory of the field $\nabla u_{\widetilde{G}}(y, y_0)$, which enters the pole y_0 $(1 < p < \infty)$. The construction of such mappings is carried out by analogy with the Green's mappings $(p = n = 3)$ considered in the monograph A.I. Januszauskas [2], as a special case of harmonic mappings with respect to M.A. Lavrentyev.

Let $f : G \to \widetilde{G}$ be p-harmonic Green's mapping of the domain G onto the domain $\widetilde{G}$, $f(x_0) = y_0$; f_t is the trace of the mapping f on the level surface $S_t(u_G)$, $J_{f_t}(x)$ is the jacobian of the trace f_t and $J_f(x)$ is the jacobian of f.

Theorem 1. *The following relations hold:* 1) $\lim\limits_{x \to x_0} J_f(x) = 1$; 2) $\lim\limits_{t \to \infty} \frac{|\nabla u_G|}{|\nabla u_{\widetilde{G}}|}\Big|_{u_G = u_{\widetilde{G}} = t} = 1$;

$$3)\ J_{f_t}(x) = \left(\frac{|\nabla u_G(x, x_0)|}{|\nabla u_{\widetilde{G}}(f(x), y_0)|}\right)^{p-1},\ x \in S_t(u_G);\quad 4)\ J_f(x) = \left(\frac{|\nabla u_G(x, x_0)|}{|\nabla u_{\widetilde{G}}(f(x), y_0)|}\right)^{p}.$$

Suppose that the domains G and $\widetilde{G}$ have regular boundaries and are homeomorphic to the ball. We denote by $R_p(x_0, G)$ the p-harmonic inner radius of the domain G with respect to the point $x_0 \in G$ (see [3]). Let $\gamma = \frac{n-p}{p-1}$.

Theorem 2. *1) If $1 < p \leqslant n$, there exists the derivative number*

$$\lambda_f^p(x_0) = \begin{cases} \lim\limits_{x \to x_0} \frac{|y - y_0|}{|x - x_0|},\ p = n; \\ \lim\limits_{x \to x_0} \left[|y - y_0|^{-\gamma} - |x - x_0|^{-\gamma}\right]^{-\frac{1}{\gamma}},\ p < n. \end{cases}$$

$$2)\quad R_p(y_0, \widetilde{G}) = \begin{cases} R_p(x_0, G),\ p > n; \\ \lambda_f^n(x_0) R_p(x_0, G),\ p = n; \\ \left[\left(R_p(x_0, G)\right)^{-\gamma} + \left(\lambda_f^p(x_0)\right)^{-\gamma}\right]^{-\frac{1}{\gamma}},\ p < n. \end{cases}$$

3) If $\widetilde{G}$ is the ball of radius R centered at y_0 then

$$J_f(x) = 1 + O(|x - x_0|^{\frac{n-1}{p-1}}),\ \textit{as } x \to x_0 \quad \textit{if and only if} \quad R_p(x_0, G) = R.$$

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MSC 31C12

On the asymptotic behavior of solutions of elliptic-type equations on non-compact Riemannian manifolds

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This work is devoted to studying the asymptotic behavior of solutions of the Poisson equation

$$\Delta u(x) - c(x)u(x) = f(x) \tag{1}$$

on Riemannian manifolds with model and quasimodel ends. In particular, we find the conditions for solvability of the Dirichlet problem with continuous boundary conditions 'at infinity'.

Let M be a complete Riemannian manifold without boundary, being such that $M = B\bigcup D_1 \bigcup \cdots \bigcup D_p$, where B is compact set, and D_i is isometric of the direct product $R_+ \times S_i$ (where $R_+ = (0, \infty)$, and S_i is a compact Riemannian manifold) endowed by a metric of the form

$$ds^2 = dr^2 + g_i^2(r)d\theta_i^2.$$

Here $g_i(r)$ is a positive smooth functions on R_+, $d\theta_i^2$ is a metric on S_i. D_i is called model end. Suppose that on D_i fulfilled $c(x) = c_i(r)$ and $f(x) = f_i(r, \theta)$.

Denoted

$$a_i(r) = \|f_i(r, \theta)\|_{L^1(S_i)},$$

$$am_i(r) = \|\Delta_\theta^m f_i(r, \theta)\|_{L^2(S_i)},$$

where $m = [\frac{3n}{4}]$, and

$$J_i = \int_{r_0}^{\infty} g_i^{1-n}(t) \left(\int_{r_0}^{t} \big(\frac{1}{g_i^2(\xi)} + c_i(\xi) + a_i(\xi) + am_i(\xi)\big) g_i^{n-1}(\xi) d\xi \right) dt,$$

where $dimM = n$.

Theorem 1. *Suppose that $J_i < \infty$ for some i. Then for any $\Phi_i \in C(S)$ and $\Psi_i \in C(S_i)$ there is unique solution of equation (1) on D_i such that $u(r_0, \theta) = \Phi_i(\theta)$ and $\lim_{r\to\infty} u(r, \theta) = \Psi_i(\theta)$.*

Theorem 2. *Suppose that $J_i < \infty$ for all i. Then for any $\Psi_i \in C(S_i)$ there is unique solution of equation (1) on M such that on each D_i*

$$\lim_{r\to\infty} u(r, \theta) = \Psi_i(\theta).$$

The proof is based on the results of the papers [1] and [2].

Similar results were obtained on manifolds with quasimodel ends.

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MSC 30E10.

Asymptotics of Hermite–Padé approximants for functions with three branch points [1]

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Let A be a finite set on the complex plane. Let $\mathcal{A}_A$ be a set of all the analytic germs at infinity $f(z) = \sum_{j=0}^{\infty} c_n z^{-n}$ such that: a) f has an analytic continuation along any path in $\mathbb{C} \setminus A$; b) the set $B(f)$ of the branch points of f is not empty: $A \supset B(f) \neq \varnothing$. For a finite set $(f_1, \dots, f_r)$ of germs $f_j \in \mathcal{A}_A$ we consider rational Hermite–Padé approximants with the common denominator $\left(\frac{P_{n,1}}{Q_n}, \dots, \frac{P_{n,r}}{Q_n}\right)$:

$$\deg Q_n \le rn, \quad (f_j Q_n - P_{n,j})(z) = O(z^{-n-1}), \quad z \to \infty, \quad j = 1, \dots, r.$$

In [1] J. Nuttall made some hypotheses describing the domains of convergence of these approximants and the limiting distribution of their poles. We consider these issues for two particular constructions in each of which the set A consists of three points.

We assume that $A := \{a_0, a_1, a_2\}$. The first construction was introduced in [2], where the Hermite–Padé approximants for the following pair (f_1, f_2) were studied:

$$f_j(z) := \int_{a_0}^{a_j} \frac{w(\zeta)d\zeta}{z - \zeta}, \quad w(\zeta) := (\zeta - a_0)^{\alpha_0}(\zeta - a_1)^{\alpha_1}(\zeta - a_2)^{\alpha_2}, \quad \alpha_j > -1.$$

In [2] the special case $A = \{-1, 0, 1\}$ was studied. The properties of the approximants for an arbitrary three-point set A follow from the results of [3].

The second construction arose in [4] where some new arithmetic properties of π were derived and the Hermite–Padé approximants for the set (f_{01}, f_{02}, f_{12}) were applied:

$$f_{jk}(z) := \int_{a_j}^{a_k} \frac{w_{jk}(\zeta)d\zeta}{z - \zeta}, \quad w_{jk}(\zeta) := (\zeta - a_j)^{n+\alpha_j}(\zeta - a_k)^{n+\alpha_k}.$$

For each of these sets, we describe the curves that attract the zeros of Q_n. We study the dependence of the topological properties of these curves from the set A. An important role in our description is played by special vector equilibrium problem for the logarithmic potential (see [5, 6]), curves with the symmetry property (S-property), and critical trajectories of quadratic differentials on algebraic curves.

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Ways of Analytic Continuation of Many-Sheeted Functions of One Variable. Applications.

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It is represented the following plan of the talk.

1. The classical Borel–Polya results on analytic continuation of Taylor–Laurent series with the help of the Borel–Laplace transform for an entire function associated with this series.

2. A variant of the Polya–Bernstein theorem on a way of analytic continuation of a Puiseux–Laurent series

$$F(p) = \sum_{k=0}^{\infty} \frac{c_k}{p^{(k+1)/\rho}}, \;\; p = (r, \varphi), \;\; r > R, \;\; \varphi \in \mathbb{R},$$

generating by the power function $z = p^{1/\rho}$, $\rho > 0$. This result is based on the Bernstein construction of a many-sheeted indicator diagram E_h of the entire function

$$f(z) = \sum_{k=0}^{\infty} \frac{c_k z^k}{\Gamma\left(\frac{k+1}{\rho}\right)}, \quad z \in \mathbb{C}$$

of order $\rho > 0$, $\rho \neq 1$ associated with the mentioned series. Here E_h is the many-sheeted surface, generated by motion of the half-plane $\Pi_h(\theta) = \{(p, \theta) : \; \Re p e^{i\rho\theta} \geq h(\theta), \; p \in \mathbb{C}\}$, where h is the indicator of f and $\theta \in \mathbb{R}$ is the motion parameter.

3. A generalization of the Borel method of analytic continuation of a Taylor series and its many-sheeted variant for a Puiseux series.

4. Properties of analytic continuation domain of the Puiseux series – expansions of the inverse to a rational function, in particular, the inverse to a Laurent polynomial

$$p = \alpha(z) := z^\rho + a_1 z^{\rho-1} + \cdots + a_n z^{\rho-n}, z \in \mathbb{C} \setminus \{0\}; \; a_n \neq 0, \;\; \rho, n \in \mathbb{N}.$$

5. A way of analytic continuation of the inverse to the polynomial $p = \alpha(z)$, $\rho = n$ and solutions of an algebraic equation $\{z \in \mathbb{C} : \; \alpha(z) = 0\}$. The problem of determination of solutions for this equation is equivalent to the problem of determination of the inverse α_n^{-1} with respect to the function $p = \alpha_n(z)$, where $\alpha_n(z) = \alpha(z) - a_n$.

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MSC 53B15, 58E25.

On connections on 3-dimensional subRiemannian Lie groups.[1]

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A subRiemannian 3-dimensional Lie group M can be considered as one of the standard 3-dimensional Lie groups occupied with 2-dimensional non-holonomic distribution $\mathcal{H}^0 \subset TM$ defining admissible directions. It was shown [1] that it possible to choose a basis (X, Y) in $\mathcal{H}^0$ such that Lie bracket will have the following form:

$$[X,Y] = Z, \quad [Y,Z] = (\chi + \kappa)X, \quad [X,Z] = (\chi - \kappa)Y. \tag{1}$$

There were several approaches to define a connection on a subRiemannian Lie group. Firstly, the Tanaka-Webster connection on M as a contact manifold. Secondly, Tanno connection on M as a CR-manifold. It was shown that equations (1) force Tanaka-Webster and Tanno connections to coincide. The third previously known connection was suggested by Wagner [2] from the absolute parallelism speculations.

We defined new type of connection assuming that the Heisenberg group should be the flattest among all 3-dimensional Lie groups. It is well-known that the nilpotentization procedure applied to a Riemannian manifold N will give its tangent space T_pN which if linear space and thus flat. The nilpotentization of any 3-dimensional subRiemannian group will give Heisenberg group. Consequently, the assumption for Heisenberg group to be the flattest is reasonable enough.

In subRiemannian case it is not trivial task to define curvature as not all directions are equivalent. We defined the holonomy flag as the family of subspaces in TM which generalizes the holonomy algebra in Riemannian case:

$$\mathcal{R}^i_{XY} = \{R(X,Y)Z \mid Z \in \mathcal{H}^i\} \bigcap \mathcal{H}^i,$$

where $\mathcal{H}^i = \{\mathcal{H}^{i-1}, \mathcal{H}^0\}$ form the standard filtration of tangent space for subRiemannian manifold M. We classified all torsion-free connections for which holonomy flag of the Heisenberg vanishes. Also it was proved that all mentioned connections on 3-dimensional subRiemannian Lie groups do not coincide.

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MSC 30C15.

Free interpolation in the class of functions of finite order in the half-plane[1]

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We denote by $[\rho, \infty]^+$ the space of functions analytic in the half-plane $\mathbb{C}_+ = \{z : \Im z > 0\}$ whose order in the sense of Govorov-Titchmarsh [1] is less than or equal to ρ, $\rho \geqslant 0$. In 1975, B. Ya. Levin and N. Wen [2] considered the problem of simple free interpolation in the space $[\rho, \infty]^+$ for $\rho > 1$. They found the necessary conditions and sufficient conditions (between which there was a difference) for its solvability in terms of the canonical Nevanlinna set of interpolation nodes. In addition, it was additionally assumed that the interpolation nodes have a single condensation point at infinity (although this also follows from sufficient conditions). The aim of this paper is to study the interpolation problem in the space $[\rho, \infty]^+$ for any $\rho \geqslant 0$. Let $A = \{a_n\}_{n=1}^{\infty} \subset \mathbb{C}_+$ be a sequence of different complex numbers, $\Lambda_n = \min\{1; \Im a_n\}$.

Definition. *The sequence A is called interpolation sequence in the space $[\rho, \infty]^+$ if for any sequence of complex numbers $\{b_n\}_{n=1}^{\infty}$ satisfying condition*

$$\sup_n \frac{\ln^+ \ln^+ |b_n|}{\ln |a_n| + 2} < \infty, \quad \limsup_{|a|_n \to \infty} \frac{\ln^+ \ln^+ |b_n|}{\ln |a_n|} \leqslant \rho\,,$$

there exist the function $F \in [\rho, \infty]^+$ such that $F(a_n) = b_n$, $n = 1, 2, \ldots$.

We formulate the main theorem of our investigation.

Theorem. *The following two statements are equivalent.*

1) *The sequence A is interpolation sequence in the space $[\rho, \infty]^+$.*

2) *Canonical product $E(z)$ of the sequence A satisfies the conditions* :

$$\sup_n \frac{1}{\ln |a_n| + 2} \ln^+ \ln^+ \frac{1}{|E'(a_n)|\Lambda_n} < \infty,$$
$$\limsup_{|a|_n \to \infty} \frac{1}{\ln |a_n|} \ln^+ \ln^+ \frac{1}{|E'(a_n)|\Lambda_n} \leqslant \rho\,.$$

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On differential properties of the mappings with s-averaged characteristic

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Example 1. *Let $D \subset \mathbb{R}^3$ — the domain, defined as following:*
$D = \left\{x \in \mathbb{R}^3; 0 < |x_1| < \infty, 0 \le x_2 \le |x_1|^\beta, 0 \le x_3 \le 1\right\}$, *where* $0 < \beta \le 2$. *Let the function* $f\colon D \to \mathbb{R}^3$, $f(x) = (y_1, y_2, y_3)$, *where* $y_1 = x_1$, $y_2 = \frac{x_2}{|x_1|+1}$, $y_3 = x_3\left(|x_1|+1\right)^{1-\alpha}$ *and* $0 < \alpha < 1$.

It is shown [1] that the introduced mapping is a mapping with s-averaged characteristic, so that the studied class off mappings is not empty and not a mapping with bounded distortion.

Theorem 1. *Let the domain $D \subset \mathbb{R}^n$, $f\colon D \to \mathbb{R}^n$, $f = (f^1, f^2, ..., f^n)$ — a mapping with s-averaged characteristic such that $f^{i_k} \in W^1_{n+\varepsilon_k, loc}(D)$, $\varepsilon_k > 0$. Let $a = \left[1 + \sum\limits_{k=1}^{n}(n+\varepsilon_k)^{-1} - mn^{-1}\right]^{-1}$, $1 \le k \le m < n$, $s > a(a-1)^{-1}$, $1 \le i_1 < i_2 < ... < i_m \le n$.*

Then $f \in W^1_{n+\delta, loc}(D)$, $\delta = n(s(a-1)-a)(s+a)^{-1}$.

Theorem 2. *Let $D \subset \mathbb{R}^n$ — a domain, $f\colon D \to \mathbb{R}^n$, $f = (f^1, f^2, ..., f^n)$ — a mapping with s-averaged characteristic such that $f^{i_k} \in W^1_{n+\varepsilon_k, loc}(D)$, $\varepsilon_k > 0$. Let $1 \le i_1 < i_2 < ... < i_m \le n$, $1 \le k \le m < n$, $s > 1$, $a = \left[1 + \sum\limits_{k=1}^{n}(n+\varepsilon_k)^{-1} - mn^{-1}\right]^{-1}$.*

Then $f \in W^1_{n+\delta, loc}(D)$, $\delta = n(s-1)(a-1)(s+a-1)^{-1}$.

In the next theorem we prove that the mappings with s-averaged characteristic are lower semicontinuous.

Theorem 3. *Let $D_m \subset D'$, $m = 0, 1, 2, ...,$ — bounded domains, $|D'| \le R < \infty$. Let $f_m\colon D \to D_m$ — a sequence of mappings with s-averaged characteristic, $s > (n-1)^{-1}$, and the sequence $\{f_m\}$ converges uniformly inside D to a continuous mapping f, $f\colon D \to \mathbb{R}^n$.*

Then

$$K_{O,s}(f) = \left(\int\limits_D K_O^s(x, f)\, d\sigma_x\right)^{1/s} \le \varliminf_{m\to\infty} K_{O,s}(f_m).$$

Theorem 4. [2] *Let $s \ge \frac{2}{n-2}$ and the mapping $f\colon D \to D^*$ is an extremal mapping in the class $K_{I,s}(D, D^*)$. Then $f \in W_2^2(D')$ for all subdomains D' such that $J(x, f) > 0$ on the closure $\overline{D'}$ and $\overline{D'} \subset D$.*

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Singular points of the Hurwitz composition

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Let the functions $p(t)$ and $q(t)$ be given by series

$$p(t) = \sum_{n=0}^{\infty} \frac{c_n}{t^{n+1}} \qquad |t| > r_p, \tag{1}$$

$$q(t) = \sum_{n=0}^{\infty} \frac{d_n}{t^{n+1}} \qquad |t| > r_q. \tag{2}$$

Then the series

$$G(t) = \sum_{n=0}^{\infty} \frac{e_n}{t^{n+1}} \qquad e_n = \sum_{k=0}^{n} C_n^k c_k d_{n-k}$$

is called the Hurwitz composition of series (1) and (2).

If the functions $p(t)$ and $q(t)$ are regular in the domains A and B, respectively ($\infty \in A$, $\infty \in B$), then, according to a well-known theorem of Hurwitz on the addition of singularities [1], the function $G(t)$ is regular in the connected component of the set $A \oplus B$, which contains a point $t = \infty$. Here $A \oplus B = (A' + B')'$, $A' + B' = = \bigcup\limits_{t_1 \in A, t_2 \in B} (t_1 + t_2)$ and A' is the complement to the domain to the whole plane. The problem of the connection between singular points of functions $p(t)$, $q(t)$ and the singularities of their Hurwitz composition has not been studied much, in contrast to the analogous question for Hadamard's composition [1].

In this paper a theorem of this type is proved.

Theorem. *Let the functions $p(t)$ and $q(t)$ be regular in the domains A and B, respectively, containing the half-plane $Re\, t > 0$, on the line $Re\, t = 0$ there are singular points of the Hurwitz composition $G(t)$ of the series (1) and (2), $t = 0$ is only one singular point of the function $p(t)$ on the line $Re\, t = 0$, $Re\, d_n \geq 0$ for $n = 0, 1, 2, \ldots$ and $\lim\limits_{n \to \infty} \frac{Re\, d_n}{|d_n|} = 1$.*

Then the point $t = 0$ is a singular point of the Hurwitz composition $G(t)$ of the series (1) and (2).

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MSC 31C12.

On solvability of a boundary value problem for the Poisson equation on unbounded open sets of Riemannian manifolds

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In this article we study questions of existence, uniqueness and belonging to given functional class of solutions for the Poisson equation

$$\Delta u = g(x), \tag{1}$$

where $g(x) \in C^{\gamma}(\Omega)$ for any subset $\Omega \subset\subset M$, $0 < \gamma < 1$ on unbounded open sets of Riemannian manifold M without boundary.

Let M be an arbitrary smooth connected noncompact Riemannian manifold without boundary and let $\{B_k\}_{k=1}^{\infty}$ be an exhaustion of M. Throughout the sequel, we assume that boundaries ∂B_k are C^1-smooth submanifolds.

Let f_1 and f_2 be arbitrary continuous functions on M, $G \subseteq M$ be unbounded set. Say that f_1 and f_2 are *equivalent on* G and write $f_1 \overset{G}{\sim} f_2$ if for some exhaustion $\{B_k\}_{k=1}^{\infty}$ of M we have $\lim_{k\to\infty} \sup_{G\setminus B_k} |f_1(x) - f_2(x)| = 0$.

It is easy to verify that the relation " $\sim$ " is an equivalence which does not depend on the choice of the exhaustion of the manifold (see also [1, 2]).

Let $\Omega \subset M$ be an arbitrary connected unbounded open set, the boundary of Ω is a C^1-smooth submanifold and $\overline{\Omega} \neq M$. If $\partial\Omega$ be non compact, then we assume that $B_k \cap \Omega$ be connected sets for all k.

Let $B \subset M$ be an arbitrary connected compact subset and the boundary of B is a C^1-smooth submanifold. Assume that the interior of B is non-empty and $B \subset B_k$ for all k.

Denote by v_B is the capacity potential of the compact set B relative to the manifold M, s.e. v_B is the harmonic function in $M \setminus B$ which satisfies to conditions $0 < v_B \leq 1$, $v_B|_{\partial B} = 1$. The manifold M is called Δ-*strict* manifold if for some compact set $B \subset M$ there is an capacity potential v_B such that $v_B \overset{M}{\sim} 0$ (see [1, 2]).

A continuous function f on M (on Ω, resp.) is called *admissible* on M (on Ω, resp.) for equation (1) if there is a solution of this equation on M (on Ω, resp.) such that $u \overset{M}{\sim} f$ ($u \overset{G}{\sim} f$, resp.).

We now formulate the main result.

Theorem. *Let M be a Δ-strict manifold, $\Omega \subset M$ be an unbounded open set with C^1-smooth boundary $\partial\Omega$, f be an admissible continuous on Ω function and ϕ be a continuous on $\partial\Omega$ function such that $\phi \overset{\partial\Omega}{\sim} f$. Then there exists unique solution u of equation (1) on Ω such that $u|_{\partial\Omega} = \phi$ and $u \overset{\partial\Omega}{\sim} f$.*

We remark that similar result for harmonic function was established early in [2].

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Decomposable operators on L_p-direct integrals [1]

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This abstract is based on the joint work with Dr. N.A. Evseev on the same topic.

In recent years spaces of Banach-valued functions are intensively studied, for example spaces $L_p(\Omega, X)$. If instead of one Banach space X one consider a family of spaces $\{X_\omega\}_{\omega\in\Omega}$, then we come to the concept of L_p-direct integral $\Big(\int_\Omega^\oplus X_\omega\, d\mu\Big)_{L_p}$ (see [1,2]). It refer us to some classical types of functional spaces, for example mixed norm Lebesgue spaces. In this paper we study the boundedness of operators acting from one L_p-direct integral to another.

Let (T,μ), (S,ν) is a measurable space and $\{W_t\}_{t\in T}$, $\{V_s\}_{s\in S}$ are associated with this spaces families of Banach spaces. Next, let $F \subset S\times T$, (F,λ) also is a measurable space. Further we define on F operator-valued function $P(s,t) : W_t \to V_s$. Each of the operators $P(s,t)$ is linear and bounded. Function P induces an operator $M_F : \Big(\int\limits_T^\oplus W_t\, d\mu\Big)_{L_p} \to \Big(\int_F^\oplus V_s\, d\nu\Big)_{L_q}$ by the rule $M_F[f](s,t) = P(s,t)[f(t)]$. Note that the operator of this type is a generalization of the concept of decomposable operators.

The main result is the following theorem:

Theorem 1. *Let λ is absolutely continuous with respect to $\mu\times\nu$, then operator M_F is bounded if and only if*

$$\|P(s,t)\| \cdot J^{\frac{1}{q}}(s,t) \in L_{\kappa,q}(F),$$

where $J(s,t) = \frac{d\lambda}{d\mu\times\nu}$ — Radon-Nikodym derivative of measure λ with respect to $\mu\times\nu$, $L_{\kappa,q}(F)$ — mixed norm Lebesgue space.

If we are going to vary the set F and operators $P(s,t)$ then we can have a significant impact on the character of the operator M_F. In particular, if F is the graph of a mapping $\varphi : \Omega \to \Omega'$ ($\Omega \subset T\times X$, $\Omega' \subset S\times Y$) and operators $P(s,t)$ are composition operators, we obtain the next result:

Theorem 2. *Measurable mapping $\varphi : \Omega \to \Omega'$ of the form $\varphi(t,x) = (\varphi_1(t), \varphi_2(t,x))$ induce a bounded composition operator $C_\varphi : L_{p_1,p_2}(\Omega') \to L_{q_1,q_2}(\Omega)$, if and only if*

$$J^{\frac{1}{q_1}}_{\varphi_1^{-1}}(s) J^{\frac{1}{q_2}}_{\varphi_2^{-1}}(s,y) \in L_{\frac{p_1q_1}{p_1-q_1},\frac{p_2q_2}{p_2-q_2}}(\Omega').$$

The same result for the case of the operator $C_\varphi : L_{p_1,p_2}(\Omega') \to L_{p_1,p_2}(\Omega)$ was obtained in recent work [3].

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MSC 30J99.

Belonging to the Mackenhoupt class of a function vanishing at the boundary of the unit circle

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Let $B_1 = \{z \in \mathbb{C} : |z| < 1\}$, Γ_0 — boundary arc of the circle B_1,

$$\Gamma_0 = \{z \in \partial B_1 : \arg z_2 \leq \arg z \leq \arg z_1\}, z_1, z_2 \in \partial B_1,$$

$$\arg z_2 < \arg z_1, \arg z_1 - \arg z_2 \leq 2\pi.$$

Lemma. *Let $dist\,(z, \Gamma_0)$ — distance from point $z \in \mathbb{C}$ to arc Γ_0, $d\,(z)$ — a non-negative continuous function in $\mathbb{C}$, that vanishes in Γ_0, positive in $\mathbb{C}\backslash\Gamma_0$, satisfying the conditions*

$$d\,(z) = d\left(\frac{1}{\bar{z}}\right), \forall z \in \bar{B}_1,$$

there exist constants $\alpha \in (0; 1)$, c_1, c_2, $0 < c_1 < c_2$, such that

$$c_1 dist^{\alpha}\,(z, \Gamma_0) \leq d\,(z) \leq c_2 dist^{\alpha}\,(z, \Gamma_0)\,,$$

$z \in B_1$, belonging to some neighborhood Γ_0. Then $d\,(z)$ belongs to the Mackenhoupt class $A_2\,(\mathbb{C})$ [1].

Co-authors obtained result was used in [2] to obtain improved properties of integrability of derivatives of a certain class of quasiconformal mappings, which are solutions of the conjugate (nonlinear) Beltrami equation.

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Continuation of Multiple Power Series in a Sectorial Domain by Means of Interpolating the Coefficients by Meromorphic Function [1]

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For multiple power series centered at the origin we consider the problem of its analytic continuability to a sectorial domain. The condition for continuability is formulated in terms of a meromorphic function that interpolates the series coefficients. For series in one variable this problem has been studied in works of E. Lindelöf, N. Arakelian, and others.

Consider a multiple power series

$$f(z)=\sum_{k\in\mathbb{N}^n} f_k z^k, \tag{1}$$

where $z^k=z_1^{k_1}...z_n^{k_n}$. We say that a function $\varphi(\zeta)$ *interpolates* the coefficients of the series (1), if

$$\varphi(k)=f_k \ \text{ for all } \ k\in\mathbb{N}^n\subset\mathbb{C}^n.$$

The complex variables ζ_j we write as $\zeta_j=\xi_j+i\eta_j$, thus ξ is a vector of the real subspace of $\mathbb{C}^n$, and η is a vector of the imaginary subspace.
We assume that on the imaginary subspace the interpolating function φ admits the estimate $|\varphi(i\eta)|\le\tilde g(\eta)$ with the piece-wise affine function

$$\tilde g(\eta)=\sum_{m=1}^{p}\varepsilon_m|a_m^1\eta_1+...+a_m^n\eta_n+a_m^0|. \tag{2}$$

Together with $\tilde g(\eta)$ we consider a piece-wise linear function

$$g(\eta)=\sum_{m=1}^{p}\varepsilon_m|a_m^1\eta_1+...+a_m^n\eta_n|+\pi\sum_{k=1}^{n}(\eta_k-|\eta_k|),\quad \varepsilon_m=\pm1$$

that has non-smooth points (points where linearly breaks) on the set of hyperplanes

$$a_m^1\eta_1+...+a_m^n\eta_n=0,\ m=1,...,p\ \text{ and }\ \eta_k=0,\ k=1,...,n.$$

These hyperplanes divide $\mathbb{R}^n$ into cones that form a fan. Denote by $\pm\mu_1,...,\pm\mu_d$ the one-dimensional generators of this fan. They define the dual (to the fan) polyhedron

$$P=\{\alpha\in\mathbb{R}^n:(\pm\mu_\nu,\alpha)\ge g(\pm\mu_\nu)\},\ \nu=1,...,d.$$

Theorem *Let a meromorphic function φ, that does not have poles in $\mathbb{R}^n_+ + i\mathbb{R}^n$, interpolate the coefficients of the series (1). If φ has a majorant (2) on the imaginary subspace and there exists $\delta>0$ and $b\in\mathbb{R}^+$ such that on $\mathbb{R}^n_+ + i\mathbb{R}^n$ the function φ satisfies*

$$\log|\varphi(re^{i\theta})|\le\sum_{j=1}^{n}\left((\pi-\delta)|\sin\theta_j|+b\cos\theta_j\right)r_j+C.$$

Then the sum of the series extends analytically into the sectorial domain $Arg^{-1}(P^o)$, where P^o is the interior of the polyhedron P.

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On a boundary correspondence for homeomorphisms with weighted bounded (p, q)-distortion [1]

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The problem of the boundary correspondence for conformal mappings is a classical result of Carathéodory and is known as the theory of Carathéodory prime ends. In the last century, the theory of the boundary correspondence in the context of classical function theory, as well for quasiconformal mappings, was thoroughly developed by Suvorov, Zorich, Nakki, Väisälä, etc.

In this talk, the authors consider the boundary correspondence problem for mappings with (θ, σ)-weighted bounded (p, q)-distortion, introduced in [1,2]. These mappings generalize mappings with bounded distortion (quasiregular mappings). In [3] it was observed that homeomorphisms $f\colon D \to D'$ with (θ, σ)-weighted bounded (p, q)-distortion, where $n - 1 < q \le p < \infty$, induce bounded composition operators $f^*\colon L^1_p(D', \sigma) \to L^1_q(D, \theta)$ of weighted Sobolev spaces if weight functions θ and σ are in Muckenhoupt classes A_q and A_p, respectively.

The idea for introducing "new boundary elements" is based on application of *capacity metric*, as discussed in [2]. The completion by this metric adds improper 'boundary' elements to the domain. That is, the set of boundary elements forms an *associated boundary.* If a homeomorphism induces a bounded composition operator on Sobolev spaces, then the inverse homeomorphism is Lipschitz in the capacity metric. Hence, it has a continuous extension onto the associated boundary. Based on these ideas, we prove that an inverse mapping to a homeomorphism $f\colon D \to D'$ with (θ, σ)-weighted bounded (p, q)-distortion, where $n - 1 < q \le p \le n$, has a continuous extension on the Euclidean boundary if the domains D and D' are locally connected on the boundaries and the support of any boundary element consists of only one point.

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Composition operators on discrete Hardy spaces of a tree

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(A joint work with Prof. S. Ponnusamy)

The study of composition operators on various analytic function spaces defined on the open unit disk is well-known. Recently, Colonna et al. defined the Lipschitz space of a infinite tree as discrete analogue of classical Bloch space and study the composition operator on them. Since infinite trees are widely regarded as discrete version of Hyperbolic disk in complex plane, studying composition operators on trees becomes interesting and it opens up the door for studying composition operators from analytic function spaces to discrete function spaces.

We have defined a discrete analogue of generalized Hardy spaces on a homogeneous rooted tree and studied the basic properties such as boundedness and compactness of composition operators on them. Also, we calculated the operator norm of the composition operator when the inducing symbol is an automorphism of a homogenous tree.

In this presentation, we first recall some basic issues concerning composition operators on Lipschitz space of a infinite tree graph and we continue the discussion about composition operators on the discrete analogue of Hardy spaces. We mainly discuss some necessary and sufficient condition on the symbol ϕ so that the composition operator C_ϕ induced by ϕ is bounded and compact operator.

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Extended Exton's Triple Hypergeometric Functions $X_{8,p}$ and Associated Bounding Inequalities

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Motivated by certain recent extensions of the Euler's beta, Gauss's hypergeometric and confluent hypergeometric functions [1], we extend Exton's triple hypergeometric function $X_{8,p}$ by making use one additional parameter in the integrand. Systematic investigation of its properties, among others various integral representations of Euler and Laplace type, Mellin transforms, Laguerre polynomial representation, transformation formulæ and a recurrence relation follow. Also, by virtue of Luke's bounds for hypergeometric functions and various bounds upon the Bessel functions appearing in the kernels of the newly established integral representations we deduce a set of bounding inequalities for the extended Srivastava's triple hypergeometric function $X_{8,p}$.

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On the quasiconformal geometry of the Neumann eigenvalue problem[1]

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We study spectral properties of the Neumann-Laplace operator in the terms of the quasiconformal geometry of domains using the notion of quasiconformal regular domains [1]. This class of domains includes Lipschitz domains, star-shaped domains and some fractal domains (snowflakes).

Our machinery is based on the geometric theory of composition operators on Sobolev spaces [2]. This study leads to the lower estimates of the first non-trivial Neumann eigenvalues in the terms of derivatives of quasiconformal mappings, because there exists connection between composition operators on Sobolev spaces and the quasiconformal mappings theory [3].

The main result is

Theorem 1. *Let $\Omega \subset \mathbb{R}^2$ be a K-quasiconformal β-regular domain. Then the spectrum of the Neumann–Laplace operator in Ω is discrete, and can be written in the form of a non-decreasing sequence:*

$$0 = \mu_0(\Omega) < \mu_1(\Omega) \leq \mu_2(\Omega) \leq \ldots \leq \mu_n(\Omega) \leq \ldots,$$

and

$$\frac{1}{\mu_1(\Omega)} \leq \frac{4K}{\sqrt[\beta]{\pi}} \left(\frac{2\beta-1}{\beta-1}\right)^{\frac{2\beta-1}{\beta}} \|J_\varphi \mid L_\beta(\mathbb{D})\|,$$

where $\varphi : \mathbb{D} \to \Omega$ is the K-quasiconformal mapping and $J_\varphi(x,y)$ is a Jacobian of mapping φ at a point (x,y).

(With joint Vladimir Gol'dshtein and Alexander Ukhlov).

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The Method Of Modules Of Curves Families In Problems On The Extremal Decomposition[1]

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We extend a classical result by Lavrent'ev concerning the product of the conformal radii of planar non-overlapping domains to the case of domains in the n-dimensional Euclidean space. The conformal radius is then replaced by the n-harmonic Levitskii radius [1] and the non-overlapping condition is replaced by a weaker geometric condition. The proofs are based on the technique of modulii of curve families. Conformal invariance of the module plays an important role in the proofs. Using the same method, we extend a classical result of Kufarev concerning the product of the conformal radii of planar non-overlapping domains in the unit disk. In addition, an inequality for n-harmonic radius of a star-shaped domain has been proved [2]. For example, we have

Theorem. *Let D_1, D_2 be domains in $\mathbb{R}^n$, $a_1 \in D_1$ $a_2 \in D_2$ and $D_1 \cup D_2$ contains no curves*

$$x(t) = a_1 + \frac{tv|a_2 - a_1| + a_2 - a_1}{|tv|a_2 - a_1| + a_2 - a_1|^2}|a_2 - a_1|^2, \quad 0 \le t \le \infty.$$

Then

$$R_n(D_1, a_1) \cdot R_n(D_2, a_2) \le |a_1 - a_2|^2.$$

Here v is fixed vector, $|v| = 1$, $R_n(D_i, a_i)$ is n-harmonic radius, $i = 1, 2$.

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(p,ω)-equivalent sets on a Riemann surface

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Let $\mathcal{R}$ be a Riemann surface glued from finitely or countably many domains in the extended complex plane $\bar{C} = C \cup \{\infty\}$ so that the following conditions are satisfied: each point in $\mathcal{R}$ projects onto a point $w = \text{pr } W$ in one on the glued domains; each point in $\mathcal{R}$ has a neighbourhood which is univalent disk or multivalent disk with the unique branch point at the centre of the disk.

The operation of projection $W \to \text{pr}W = w$ induces the Lebesgue 2-dimensional measure σ, the Hausdorff 1-dimensional measure $\mathcal{H}^1$, the Euclidean metric ds and the spherical metric dh on $\mathcal{R}$ (see [1]).

Let G be an open set in $\mathcal{R}$ such that $\bar{G}$ is a compact in $\mathcal{R}$. Set $\mathcal{R}_\infty = \{W \in \mathcal{R} : \text{pr}W = \infty\}$, $\mathcal{R}_b = \{W \in \mathcal{R} : W \text{ is a ramification point}\}$. Denoted by $\mathcal{D}_{p,\omega}(G)$ the class of functions u which are continuous in G and are locally Lipschitz in G and are constant in some neighborhood of each point of $G \cap \big(\mathcal{R}_\infty \cup \mathcal{R}_b\big)$ for which

$$\|u\| = \|u\|_{p,\omega}(G) = \Big(\int\limits_G |u|^p\, \omega\, d\sigma\Big)^{\frac{1}{p}} = \Big(\int\limits_{G\setminus(\mathcal{R}_\infty \cup \mathcal{R}_b)} |u|^p\, \omega\, d\sigma\Big)^{\frac{1}{p}} < \infty,$$

where $\omega : \mathcal{R} \to (0, +\infty)$ satisfies the A_p-condition of Muokenhoupt on $\mathcal{R}$ (see [2]).

The closure of $\mathcal{D}_{p,\omega}(G)$ in the norm $\|\cdot\|$ is denoted $L^1_{p,\omega}(G)$.

Let $E \subset G$ - relatively closed set in G such that $E \cap (\mathcal{R}_\infty \cup \mathcal{R}_b) = \emptyset$ and for each point $X \in E$ there exists a univalent neighborhood $O(X)$, for which closure $\text{pr}\big(O(X) \cap E\big)$ is a $NC_{p,\omega}$-set on $\mathbb{R}^2$ (see [3]), then E is called a $NC_{p,\omega}$-set in G.

Let D — be an open set in $\mathcal{R}$, not necessarily with a compact closure in $\mathcal{R}$. Two open sets D_1 and D $(D_1 \subset D)$ will be called (p,ω)-equivalent, if the restriction operator $\theta : L^1_{p,\omega}(D) \to L^1_{p,\omega}(D_1)$ $\big(\theta u = u|_{D_1}, u \in L^1_{p,\omega}(D)\big)$ is an isomorphism of vector spaces $L^1_{p,\omega}(D)$ и $L^1_{p,\omega}(D_1)$.

Theorem In order for the open sets D, $D_1 \subset \mathcal{R}$, where $D_1 \subset D$, $(D \setminus D_1) \cap (\mathcal{R}_\infty \cup \mathcal{R}_b) = \emptyset$ be (p,ω)-equivalent on $\mathcal{R}$, it is necessary and sufficient that the $D \setminus D_1$ be $NC_{p,\omega}$-set on D.

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On the Carleman formula for a matrix ball of the third type

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Let $\mathbb{C}[m \times m]$ be the space of $[m \times m]$ matrices with complex elements. The direct product of n instances of $\mathbb{C}[m \times m]$ is denoted by $\mathbb{C}^n[m \times m]$.

The set

$$B^{(1)}_{m,n} = \{Z = (Z_1, ..., Z_n) \in \mathbb{C}^n[m \times m] : I^{(m)} - \langle Z, Z \rangle > 0\},$$

where $\langle Z, Z \rangle = Z_1Z_1^* + Z_2Z_2^* + \cdots + Z_nZ_n^*$ is the ¡¡scalar¿¿ product I is the unit $[m \times m]$ matrix $Z_\nu{}^* = \bar{Z}'_\nu$ is the matrix conjugate and transposed to Z_ν, $\nu = 1, 2, ..., n$, is called a matrix ball (of the first type) (see [1]). Here $I^{(m)} - \langle Z, Z \rangle > 0$ means that the Hermitian matrix is positive definite; all eigenvalues are positive.

The matrix ball $B^{(3)}_{m,n}$ of the third type (see [4]):

$$B^{(3)}_{m,n} = \left\{Z = (Z_1, ..., Z_n) \in \mathbb{C}^n[m \times m] : I^{(m)} + \langle Z, Z \rangle > 0,\ Z'_\nu = -Z_\nu,\ \nu = 1, ..., n\right\}.$$

We denote the boundary of the Shilov boundary by the matrix ball $B^{(3)}_{m,n}$ by $X^{(3)}_{m,n}$, that is,

$$X^{(3)}_{m,n} = \left\{Z = (Z_1, ..., Z_n) \in \mathbb{C}^n[m \times m] : I^{(m)} + \langle Z, Z \rangle = 0,\ Z'_\nu = -Z_\nu,\ \nu = 1, ..., n\right\}.$$

The objective of this paper is to construct the Carleman formula for functions of the class $H^1\left(B^{(3)}_{m,n}\right)$.

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Removability results for subharmonic functions, for harmonic functions and for holomorphic functions

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Blanchet has shown that a $\mathcal{C}^2$ subharmonic function can be extended through a $\mathcal{C}^1$ hypersurface provided the function is continuous throughout and satisfies a certain $\mathcal{C}^1$-type continuity condition on the exceptional hypersurface. Later we improved Blanchet's result, at least in a certain sense, measuring the exceptional set with the aid of Hausdorff measure. With the aid of this result, we gave certain extension results for harmonic and for holomorphic functions, related to Besicovitch's and Shiffman's well-known extension results, at least in some sense. Now we return to this subject. First, we refine our subharmonic function extension result slightly still more. Though our result might be considered a little bit technical and even complicated, it is, nevertheless, flexible. Second and as an example of its flexibility, we give, among others and as a corollary, a new concise extension result for subharmonic functions:

Corollary. *Suppose that Ω is a domain in $\mathbb{R}^n$, $n \geq 2$. Let $E \subset \Omega$ be closed in Ω and let $\mathcal{H}^{n-1}(E) = 0$. Let $u : \Omega \setminus E \to \mathbb{R}$ be subharmonic and such that the following conditions hold:*
(i) $u \in \mathcal{L}^1_{\mathrm{loc}}(\Omega)$, *(ii)* $u \in \mathcal{C}^2(\Omega \setminus E)$, *(iii) for each* j, $1 \leq j \leq n$, $\frac{\partial^2 u}{\partial x_j^2} \in \mathcal{L}^1_{\mathrm{loc}}(\Omega)$.
Then u has a subharmonic extension to Ω.

Moreover and with the aid of our subharmonic extension result, we slightly improve our previous extension results for harmonic and for holomorphic functions given in [8,9]. In addition, we recall a slightly related extension result for holomorphic functions.

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Comparable functionals of convex domains[1]

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Let G be a convex plane domain. Denote by $\mathbf{P}(G)$ a torsional rigidity of a domain, by $\mathbf{I}_p(G)$ p–order Euclidean moment of G with respect to its boundary, and by $\boldsymbol{\rho}(G)$ the inradius of G, i. e. $\boldsymbol{\rho}(G) := \sup\{\rho(x,G) : x \in G\}$, where $\rho(x,G)$ is the distance function a point x to the boundary ∂G.

Theorem. *Functionals* $\mathbf{P}(G)$ *and* $\mathbf{I}_p(G)\boldsymbol{\rho}(G)^{2-p}$ $(p \geq -1)$ *are comparable quantities in the class of convex domains in a sense of Pólya and Szegö.*

In the report we will show estimates of the exact constants of the ratios of functionals as a function of p. Also we will present generalized inequalities with additional term. We note that in [1] was proved the same assertion in the class of simply connected domains, but only with $p = 2$.

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Riemann - Hilbert boundary value problem with several points of turbulence [1]

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Let D be the unit disk in the complex plane $z = re^{i\theta}$, $L = \partial D$. We consider the Riemann–Hilbert boundary-value problem for analytic functions $\Phi(t)$ with boundary condition

$$\Re[e^{-i\nu(\theta)}\Phi(t)] = \frac{c(t)}{|G(t)|}, \quad t = e^{\theta},$$

here the set function $G(t) \neq 0$ and $|G(t)|$ satisfies the Hölder condition at all points of L, and function $\nu(\theta) = arg\, G(t)$, $t = e^{i\theta}$, satisfies the Hölder condition at all points of L except finite number of points t_j, $j = \overline{1,n}$, where it has essential discontinuities.

Thus, the problem under consideration belongs to boundary-value problems with undefined index.

We assume that for $t = e^{i\theta}$ there is valid representation

$$\nu(\theta) = \sum_{j=1}^{n} \nu_j(\theta) + \widetilde{\nu}(\theta), \quad \nu_j(\theta) = \begin{cases} \nu_j^-|\sin((\theta-\theta_j)/2)|^{-\rho_j}, & 0 \le \theta < \theta_j, \\ \nu_j^+|\sin((\theta-\theta_j)/2)|^{-\rho_j}, & \theta_j < \theta \le 2\pi, \end{cases}$$

where ν_j^+, ν_j^-, ρ_j are known values, $0 < \rho_j < 1$, and function $\widetilde{\nu}(\theta)$ satisfies the Hölder condition on $[0, 2\pi]$. The function $\widetilde{\nu}(\theta)$ satisfies inequality

$$\widetilde{\nu}(2\pi) - \widetilde{\nu}(0) = \sum_{j=1}^{n} (\nu_j^- - \nu_j^+)|\sin(\theta_j/2)|^{-\rho_j}.$$

We obtained formulas for its general solution, investigate existence and uniqueness of solutions, and describe the set of solutions in the case of non-uniqueness.

The given work continues the researches published in our article [1].

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On Yamashita's conjecture for some classes of univalent functions

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Abstract: Let $f(z)$ be a function analytic in the unit disk $\mathbb{D}$. For $0 < r \leq 1$, we denote by $\Delta(r, f)$, the area of the image of the subdisk $\mathbb{D}_r = \{z : \ |z| < r\}$ under $f(z) = \sum_{n=0}^{\infty} a_n z^n$. Thus,

$$\Delta(r, f) = \iint_{\mathbb{D}_r} |f'(z)|^2 = \pi \sum_{n=1}^{\infty} n|a_n|^2 r^{2n}\, dxdy \quad (z = x + iy).$$

Computing this area is known as the *area problem for the function of type f.* We call f a Dirichlet-finite function if $\Delta(1, f)$, the area covered by the mapping $z \to f(z)$ for $|z| < 1$, is finite, and in this case, we say that f has finite Dirichlet integral. In 1990, Yamashita conjectured that $\Delta(r, z/f) \leq \pi r^2$ for convex functions f and it was finally settled in 2013 by Obradović *et al.*

Denote by $\mathcal{F}$ a subclass of the class of normalized univalent and analytic functions in $\mathbb{D}$. In this talk, we present the extremal problem for the Yamashita functional

$$\max \Delta \left(r, \frac{z}{f(z)} \right)$$

when $f \in \mathcal{F}$. In particular, this discussion includes the solution of the Yamashita conjecture for the class $\mathcal{S}^*(A, B)$ defined by a subordination relation, which was suggested in [2] and partially it is solved in [3].

This talk is based on the following references.

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On doubling formula for Gamma function. [1]

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One considers the following functional equations for functions $f(z)$ of complex variable defined over the unit interval $(0,1)$:

$$f\left(\frac{z}{q}\right)+f\left(\frac{z}{q}+\frac{1}{q}\right)+f\left(\frac{z}{q}+\frac{2}{q}\right)+\cdots+f\left(\frac{z}{q}+\frac{(q-1)}{q}\right)=q^{\alpha}f(z).$$

The equations of this type for functions of real variable were first time investigated by E. Artin [1] in connection with characterization of Gamma function by Legendre doubling and Gauss multiplication formulas. Bernoulli polynomial satisfy these equations for every natural q and negative integral α. For natural $\alpha>1$ these equations has as a solution so called polygamma function $\psi^{(\alpha)}(z)$ (where $\psi(z)$ denotes the logariphmic derivative of the Gamma function)

Theorem 1. *Every meromorphic function, satisfying the functional equation*

$$f\left(\frac{z}{2}\right)+f\left(\frac{z}{2}+\frac{1}{2}\right)=2^{\alpha}f(z)$$

for a natural $\alpha>1$, is a linear combination of the polygamma function $\psi^{(\alpha-1)}(z)$ and its complement $\psi^{(\alpha-1)}(1-z)$.

Theorem 2. *The general solution of the functional equation*

$$f\left(\frac{z}{2}\right)+f\left(\frac{z}{2}+\frac{1}{2}\right)=2f(z)$$

for meromorphic functions is

$$a\operatorname{ctg}(\pi z)+b$$

where a and b are arbitrary constant.

Theorem 3. *Gamma function is the only meromorphic function satisfying Legendre equation:*

$$\Gamma\left(\frac{z}{2}\right)\Gamma\left(\frac{z}{2}+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{z-1}}\Gamma(z),$$

and such that $\Gamma(1)=1$.

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MSC 49J40.

On Equilibrium of pending Drop lacking axial Symmetry

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Early we had considered equilibrium of pending axisymmetrical drop taking into account flexural rigidity of its intermediate layer. We consider now the same problem for the drops lacking axial symmetry. First of all, we prove that any continuously differentiable surface with positively determined first quadratic form admits almost global half-geodesic parameterization. In the general setting this problem stands to be open. In order to prove this result we reduce it to the problem of existence of generalized solution of non-linear Beltrami equation. This equation admits degeneration on unknown set with unknown velocity. Using quasiconformal mappings corresponding to linear Beltrami equations approximating non-linear one we construct its generalized solution, which permits to detect non-intersecting geodesic lines covering surface up to the set of null Hausdorff measure. On the basis of this result we deduce general form of the functional defined on continuously differentiable surfaces whose first variation yields their Gauss curvature and which is a generalization of the similar functional defined earlier by first of the authors in the axisymmetrical case. Variational problem is formulated and its solvability proved.

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The homogeneous convolution equation in space of holomorphic functions

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The problem of spectral synthesis in spaces of analitical functions is closely related with solution of the homogeneous convolution equations [1]. Here we describe appearance of the equations. Let Ω_0, Ω be the 1-connected domains given on the complex plane $\mathbf{C}$ and let U be an open disk with origin as a centre. Suppose that $\Omega_0+U \subseteq \Omega$ and that spaces of analitical functions $O(\Omega_0)$, $O(U)$, $O(\Omega)$ and $O(\mathbf{C})$ are equipped with topology of uniform convergence on compact sets. The translation operator $T_h : f(z) \to f(z+h)$ (Where $h \in U$ is shift) takes on space $O(\Omega)$ to space $O(\Omega_0)$ and it is continuous. The operator T_h naturally equals to differential operator

$$e^{hD} : f(z) \to \sum_{n=0}^{\infty} \frac{h^n}{n!}(D^n f)(z).$$

This operator's characteristic function is equal to $e^{h\lambda}$, that is

$$T_h(e^{\lambda z}) = e^{\lambda z} e^{h\lambda}.$$

We choose continuous linear operator A, which acts on space of entire functions $O(\mathbf{C})$. The differential operator (which is denoted by AT_h) is termed *A-shift operator* if it acts on $O(\Omega)$ to $O(\Omega_0)$ and it is continuous. $A(e^{h\lambda})$ is characteristic function of this operator. And so for A-shift operator AT_h we have

$$AT_h(e^{\lambda z}) = e^{\lambda z} A(e^{h\lambda}).$$

Choose arbitrary A-shift operator AT_h, function $f \in O(\Omega)$ and continuous linear functional S on space $O(\Omega_0)$. There is the function $\varphi(h) := \langle S, AT_h(f)\rangle$ which we call *A-convolution of function f and functional S*. For fixed S and U there is the operator $f \to \langle S, AT_h(f)\rangle$ which we call *A-convolution operator*, if it acts on $O(\Omega)$ to $O(U)$ and is continuous. Exponential polynomials

$$\langle S, AT_h(f)\rangle = 0, \ f \in O(\Omega),$$

are solutions of *homogeneous A-convolution equation* We denominate these *elementary solutions* of this equation.

In the report we will consider sufficient conditions for *approximation theorem*: Let Ω be a convex domain. An arbitrary solution f of homogeneous A-convolution equation can be approximated with its elementary solutions in topology of space $O(\Omega)$.

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BV-Functions and the weighted modules and capacities on a Riemann surface

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Let $\mathcal{R}$ be a Riemann surface glued from finitely or countably many domains in the extended complex plane $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ so that the following conditions are satisfied: each point in $\mathcal{R}$ projects onto a point $w = pr\ W$ in one on the glued domains; each point in $\mathcal{R}$ has a neighbourhood which is univalent disk or multivalent disk with the unique branch point at the centre of the disk.

We study elementary properties of functions of bounded variation and sets of finite perimeter in an open set $Q \subset \mathcal{R} \setminus \mathcal{K}$, where $\mathcal{K} = \{W \in \mathcal{R} : W$ is a branch point or $pr\ W = \infty\}$.

Further, using Ziemer's technique, we obtain the main result

$$C_{p,\omega}(F_0, F_1, G)^{1/p} M_{q,\omega^{1-q}}(F_0, F_1, G)^{1/q} = 1.$$

Here $p \in (1; +\infty)$ and $\frac{1}{p} + \frac{1}{q} = 1$; ω is a weight in the Muckenhoupt A_p class on $\mathcal{R}$ (see [1]); G is an open set with the compact closure on $\mathcal{R}$; F_0 and F_1 are disjoint compact sets in the closure of G; $C_{p,\omega}(F_0, F_1, G)$ is the (p, ω)-capacity of a condenser (F_0, F_1, G) (see [1]), $M_{q,\omega^{1-q}}(F_0, F_1, G)$ is the (q, ω^{1-q})-module of the family of all sets that separate F_0 from F_1 in G and do not intersect with $\mathcal{K}$.

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MSC 31C12

Asymptotic behavior of harmonic functions on model Riemannian manifolds

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This is a study of the asymptotic behavior of harmonic functions on model Riemannian manifolds with compact boundary.

Consider a Riemannian manifold of the type $M = B \cup D$, where B is precompact with non-empty interior, and D is isometric to the direct product $[r_0; +\infty) \times S$ (where $r_0 > 0$, S - closed Riemannian manifold) with metric

$$ds^2 = dr^2 + g^2(r)d\theta^2.$$

Here $g(r)$ is positive, smooth and monotonic function on $[r_0; +\infty)$; $d\theta^2$ is a metric on S.

Define

$$J = \int_{r_0}^{\infty} \frac{dt}{g^{n-1}(t)} \int_{r_0}^{t} g^{n-3}(z)\, dz.$$

In paper of Losev A.G. was proved the following statement.

Theorem 1 [1]. *Let the manifold D be such that $J < \infty$. Then for any functions $\phi(\theta) \in C(S)$ and $\psi(\theta) \in C(S)$, a unique function $u(r,\theta)$ harmonic on D is exist, such that*

$$u(r_0, \theta) = \phi(\theta)$$

and

$$\lim_{r\to\infty} \|u(r,\theta) - \psi(\theta)\|_{C(D\setminus B(r))} = 0.$$

This is a classical formulation of the Dirichlet problem, but in a number of studies the asymptotic behavior of not only harmonic functions, but also their derivatives, is studied. Some of these issues are described in the classical literature, for example in Mikhlin's book "Linear Partial Differential Equations" [2]. There is a natural interest in obtaining a similar result in the C^k.

In this paper were found conditions for the unique solvability of the following problem.

Theorem 2. *Let the manifold D be such that $J < \infty$ and $g''(r) > 0$. Then for any functions $\phi(\theta) \in C^\infty(S)$ and $\psi(\theta) \in C^\infty(S)$, a unique function $u(r,\theta)$ harmonic on D is exist, such that*

$$u(r_0, \theta) = \phi(\theta)$$

and

$$\lim_{r\to\infty} \|u(r,\theta) - \psi(\theta)\|_{C^1(D\setminus B(r))} = 0.$$

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On integral representation formula via the heat kernal

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Let $t \in \mathbb{R}$ and $x \in \mathbb{R}^n$. We define $\mathcal{L} = \partial_t - \Delta$, where $\Delta = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$. Let $T > 0$ be a fixed number. Consider a function $u(t, x)$, $t \in [0, T)$, $x \in \mathbb{R}^n$. The classical result [1, chpt 6, §1, (9)] is as follows:

Proposition. *Let $u \in C^{1,2}(0 < t < T) \cap C(0 \leqslant t < t)$ be a bounded in the strip $\{0 < t < T\}$ function such that $\mathcal{L}u$ is also a bounded function in the same strip $\{0 < t < T\}$. Then for any point $(t, x) \in (0, T) \times \mathbb{R}^n$ we have*

$$u(t,x) = \int_{\mathbb{R}^n} u(0,\xi)U(t,x-\xi)d\xi + \int_0^t \int_{\mathbb{R}^n} \mathcal{L}u(\tau,\xi)U(t-\tau,x-\xi)d\xi. \tag{1}$$

Here

$$U(t,x) = \begin{cases} \frac{\exp(-\frac{|x|^2}{4t})}{(2\sqrt{\pi t})^n}, & t > 0; \\ 0, & t \leqslant 0. \end{cases}$$

is the standard heat kernel.

The assumption on smoothness and boundedness in the proposition, unfortunately, are too heavy for the proposition to be directly applicable in some cases. For example, the standard bootstrapping argument for the classical Burgers equation $u_t - \Delta u = uu_x$ to obtain smoothness properties of solutions for merely bounded initial states [2], needs an extra justifications. Therefore it is natural to relax assumptions in the proposition above.

Let a continuous function $u : (0, T) \times \mathbb{R}^n$ satisfy the following conditions:

1. The Sobolev partial derivatives $\frac{\partial u}{\partial t}$, $\frac{\partial u}{\partial x_i}$ and $\frac{\partial^2 u}{\partial x_i^2}$ exist and belong to L^2_{loc} space;
2. $\int_0^T \operatorname*{ess\,sup}_{x \in \mathbb{R}^n} |\mathcal{L}u| dt < \infty$;
3. $\exists u_0 \in L^1_{loc}(\mathbb{R}^n)$ s.t. $u_0 = \lim_{t \to 0+} u(t, \cdot)$; in the sense of $L^1_{loc}(\mathbb{R}^n)$;
4. $\exists C < +\infty$ s.t. $\forall x_0 \in \mathbb{R}^n$ we have $\int\limits_{|x-x_0|<1} |u_0(x)| dx < C \exp\left(\frac{|x_0|^2}{4T}\right)$;
5. there exists a nonnegative function $c : (0, T) \to \mathbb{R}$ with a finite integral $\int_0^T c(t)dt < \infty$, such that $\forall x_0 \in \mathbb{R}^n$ we have $\int\limits_{|x-x_0|<1} |u(t, x)| dx < c(t) \exp\left(\frac{|x_0|^2}{4T}\right)$.

Theorem. *Under assumptions 1. — 5. the integral representation (1) holds true.*

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On the degree of convergence of Hermite – Padé approximants of Mittag – Leffler functions [1]

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Consider a set of integer functions

$$F_\gamma^j(z) = {}_1F_1(1,\gamma;\lambda_j z) = \sum_{p=0}^{\infty} \frac{\lambda_j^p}{(\gamma)_p} z^p, \quad j = 1, 2, \ldots, k, \tag{1}$$

where $\gamma \in \mathbb{C} \setminus \{0, -1, -2, \ldots\}$, $(\gamma)_0 = 1$, $(\gamma)_p = \gamma(\gamma+1)\ldots(\gamma+p-1)$ – Pochhammer symbol, $\{\lambda_j\}_{j=1}^k$ – roots of equation $\lambda^k = 1$. We can easily notice that the functions (1) it Mittag – Leffler functions. For vector-function $\overrightarrow{F_\gamma} = \{F_\gamma^1(z), F_\gamma^2(z), \ldots, F_\gamma^k(z)\}$ uniquely defined (see [1]) rational fractions (they are called diagonal type II Hermite – Padé approximants)

$$\pi_{kn,kn}^j(z) = \pi_{kn,kn}^j(z; \overrightarrow{F_\gamma}) = \frac{P_{kn}^j(z)}{Q_{kn}(z)}, j = 1, 2, \ldots, k.$$

Polynomials $Q_{kn}(z)$, $P_{kn}^j(z)$, $j = 1, 2, \ldots, k$; $\deg Q_{kn} \le kn$, $\deg P_{kn}^j \le kn$, which are in the numerator and denominator of the fraction $\pi_{kn,kn}^j(z)$, satisfy the condition

$$Q_{kn}(z)F_\gamma^j(z) - P_{kn}^j(z) = A_j z^{kn+n+1} + \ldots .$$

In [1] was proved that for $n \to +\infty$ the fractions $\pi_{kn,\,kn}^j(z; \overrightarrow{F_\gamma})$ uniformly converge to $F_\gamma^j(z)$ on compact set in $\mathbb{C}$. The following theorem described degree of this converge and thus complement the result obtained earlier by other authors (see [1]–[4]).

Theorem. *For any fixed z and $n \to +\infty$*

$$F_\gamma^j(z) - \pi_{kn,kn}^j(z; \overrightarrow{F_\gamma}) = (-1)^n x_0^{\gamma-1} \lambda_j^{n+1} B_n \times$$

$$\times \frac{z^{kn+n+1}}{(\gamma)_{kn+n}} e^{\lambda_j(1-x_0)z} e^{\frac{\sum_{i=1}^k \lambda_i}{k+1} z} \left(1 + O(1/n)\right), \ j = 1, 2, \ldots, k,$$

where

$$x_0 = \sqrt[k]{\frac{1}{k+1}}, \ B_n = \sqrt{\frac{2\pi}{n\sqrt[k]{(k+1)^{k+2}}}} \left(\frac{k}{\sqrt[k]{(k+1)^{k+1}}}\right)^n .$$

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Uniqueness sets of the simple-layer potential for the Helmholtz equation.[1]

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We consider the Dirichlet problem for the Helmholtz equation:

$$\begin{cases} \Delta u + k^2 u = 0 \text{ в } Q, \\ u\Big|_{\partial Q} = \varphi, \ \varphi \in L_2(\partial Q), \end{cases}$$

where k is a positive constant, $Q \subset \mathbb{R}^n$ is a bounded domain with Lyapunov boundary. Let $E : \mathbb{R}^n \setminus \{0\} \to \mathbb{C}$ be a fundamental solution of the Helmholtz equation.

Definition 1. *Let $\{z_i\}_{i=1}^{\infty}$ be a countable subset of the set $\mathbb{R}^n \setminus \overline{Q}$. The set of functions $\omega_i(x) = E(x - z_i)$, $i = \overline{1, \infty}$ is called a fundamental solutions set with respect to the set of singular points $\{z_i\}_{i=1}^{\infty}$.*

We are looking for an approximate solution of the following form $u^N = \sum_{i=1}^{N} c_i^N \omega_i$. Here the coefficients $c_i^N \in \mathbb{C}, i = \overline{1, N}$ can be found from the problem of minimizing the function $F(c_1^N, \ldots, c_N^N) = \left\|\varphi - \sum_{i=1}^{N} c_i^N \omega_i\right\|^2_{L_2(\partial Q)}$. This approach is known as fundamental solutions method.

Fundamental solutions method converges for any boundary condition φ if and only if the fundamental solutions set $\{\omega_i\}_{i=1}^{\infty}$ is complete in $L_2(\partial Q)$. The existence of such fundamental solutions sets was first shown in the works of V. D. Kupradze, see, e.g. [1].

Definition 2. *A set $A \subset \mathbb{R}^n \setminus \overline{Q}$ is called a uniqueness set for a single-layer potential if the validity of identity*

$$\int_{\partial Q} \rho(y) E(y - x) dy = 0$$

for all $x \in A$ implies its validity for all $x \in \mathbb{R}^n \setminus \overline{Q}$.

In this paper we show that a fundamental solutions set $\{\omega_i\}_{i=1}^{\infty}$ is complete if and only if the singular points set $\{z_i\}_{i=1}^{\infty}$ is the uniqueness set of a single-layer potential for the Helmholtz equation.

A number of simple sufficient conditions of completeness similar to those obtained in [2, 3] for the Laplace equation are given.

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Elementary solutions of homogeneous q-sided convolution equation

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Let Ω_0, Ω be convex domains on the complex plane $\mathbf{C}$ and for $\varepsilon > 0$ let $U_\varepsilon = \{z : |z| < \varepsilon\}$ be a disc. Choose arbitrary $q \in \mathbf{N}$ and let $a_0, ..., a_{q-1}$ be set of complex numbers which are not all equal to zero. And so $\omega_q = \exp \frac{2\pi i}{q}$.

Assume that A is a continuous linear operator which acts on $O(\mathbf{C})$ and $g(\lambda) \to \sum_{k=0}^{q-1} a_k g(\omega_q^k \lambda)$; AT_h is a continuous linear operator which acts on $O(\Omega)$ to $O(\Omega_0)$ and $f(z) \to \sum_{k=0}^{q-1} a_k f(z + \omega_q^k h)$. We call $AT_h : O(\Omega) \to O(\Omega_0)$ *q-sided shift operator* ($h \in U_\varepsilon$ is a shift).

Choose arbitrary q-sided shift operator $AT_h : O(\Omega) \to O(\Omega_0)$, function $f \in O(\Omega)$ and continuous linear functional S which acts on $O(\Omega_0)$. We call $\psi_A(h) := \langle S, AT_h(f)\rangle$ *q-sided convolution of function f and functional S*. For fixed S and ε linear operator $f \to \psi_A(h) := \langle S, AT_h(f)\rangle$ is termed *q-sided convolution operator.*

q-sided convolution operator acts on $O(\Omega)$ to $O(U_\varepsilon)$ and it is continuous. Moreover $f \to \langle S, AT_h(f)\rangle$ and D^q are commutative. The kernel W_S of q-sided convolution operator $f \to \langle S, AT_h(f)\rangle$ is a closed D^q-invariant subspace of $O(\Omega)$ [1].

An equation of form
$$\langle S, AT_h(f)\rangle = 0, \ f \in O(\Omega),$$
is termed *homogeneous q-sided convolution equation.* This equation's space of solutions W_S is a D^q-invariant subspace of $O(\Omega)$.

We call exponential polynomial *an elementary solution* if it satisfies (1). Function $\varphi(\lambda) := \langle S, \exp \lambda z\rangle$ is termed characteristic function of S. This function is an entire function. And it is of exponential type.

Proposition. *If λ_0 is a zero of function $\varphi(\lambda)$ and its multiplicity equals n exponential polynomials*
$$\exp \lambda_0 z, \ z \exp \lambda_0 z, ..., z^{n-1} \exp \lambda_0 z$$
and arbitrary linear combinations of them are elementary solutions of equation (1).

This proposition gives just a part of all the equation's (1) elementary solutions. In the report we will describe the set of equation's (1) solutions.

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MSC 26B15

Estimate of a polynomial characteristic of a multidimensional domain satisfying the inner cone condition

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This property will allow us to obtain an estimate for the rate of uniform convergence to the exact solution. We shall also make use of the following polynomial characteristics of a domain $\Omega \subset R^n$:

$$\lambda_N = \inf_P \frac{\left(\int\limits_\Omega |\nabla P|^2 \, dx \right)^{1/2}}{\sqrt{|\Omega|} \sup\limits_\Omega |\nabla P|},$$

where the infinum is taken over all polynomials of degree at most N in each variable. It is clear that $0 < \lambda_N \le 1$. We shall estimate the rate of convergence of λ_N to zero as $N \to \infty$. We note, that this characteristic of the domain is used in estimating the rate of convergence of polynomial solutions of the minimal surface equation. We have obtained the lower bound for λ_N in the case when is a cube with side $a > 0$. We observe that this estimate is independent of the size of cube.

$$\lambda_N \ge \frac{1}{2^{n+1}} \frac{\sqrt{\omega_n}}{2^{n/2} N^n \sqrt[4]{n^n}}, \tag{1}$$

which made it possible to estimate the value λ_N for domain Ω, we shall assume that $\Delta(\Omega) > 0$, where

$$\Delta(\Omega) = \inf_{z_0 \in \Omega} a(z_0)$$

and $a(z_0)$ is defined as follows: for each $z_0 \in \Omega$ there exists a maximal cube $K(z_0) \subset \Omega$, with sides not necessarily parallel to the axes such that $z_0 \in K(z_0)$. Let the side of the cube be $a(z_0)$. Suppose that we are given a domain $\Omega \in R^3$. We denote by $\beta(\Omega) \in (0, \pi/2]$, an angle such that each point z_0 of the domain is contained in the parallelepiped $R \subset \Omega$ with an acute angle $\beta(\Omega)$. For each $z_0 \in \Omega$ we find a parallelepiped $R \subset \overline{\Omega}$ with maximal side such that $z_0 \in D$. Let the side of this parallelepiped $h(z_0) > 0$. We let

$$H(\Omega) = \inf_{z_0 \in \Omega} h(z_0) > 0.$$

Arguing as [1], we obtain the inequality:

Theorem. *Let a bounded domain $\Omega \subset R^3$ be such that $H(\Omega) > 0$ and $\beta(\Omega) > 0$. Then the following estimate*

$$\lambda_N \ge \sqrt{\frac{\pi}{|\Omega|}} \frac{H(\Omega) \cos \beta}{96 \sqrt[4]{3} N^3 \sqrt{1 + 2 \sin^2 \beta}}.$$

holds true.

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The Main Boundary Value Problem of the Membrane Theory of Convex Shells for One Class of Symmetric Domes

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In the view of membrane theory of convex shells [1] the next problem is under consideration. Let S be a simply connected median surface of the thin elastic shell. Let S be an interior part of the ovaloid $S_0 \in W^{3,p}$, $p > 2$, of strictly positive Gaussian curvature. The boundary $L = \partial S$ is piecewise-smooth, contains the corner points c_j, and consisting with finitely many arcs $L_j \in C^{1,\varepsilon}$, $0 < \varepsilon < 1$, $j = 1, 2, ..., n$. We study the realization problem of the momentless stressed balance of this thin shell. Let us introduce the following notation: $\hat{J}$ is the mapping of the surface S to the complex plane $\zeta = u^1 + iu^2$ defined by the choice of a conjugate isometric parametrization (u^1, u^2) on the surface S, $D = \hat{J}(S)$ is a bonded domain in the ζ-plane with boundary $\Gamma = \hat{J}(L)$. Γ contains the corner points $\zeta_j = \hat{J}(c_j)$. By [1] our boundary value problem is reduced to finding in D a generalized analytic function $\Omega(\zeta)$ satisfying the boundary condition

$$\operatorname{Re}\{[s^1(\zeta) + is^2(\zeta)][\beta(\zeta)(t^1(\zeta) + it^2(\zeta)) - \alpha(\zeta)(s^1(\zeta) + is^2(\zeta))]\Omega(\zeta)\} = g(\zeta), \quad (1)$$

where $\zeta \in \Gamma$, $s^k, k = 1, 2$, are the coordinates of the tangent to Γ unitary vector, $t^k, k = 1, 2$, are the coordinates of unitary vector $\vec{t}$ of t direction on the plane, being the $\hat{J}$ - image of τ direction on the surface S_0 orthogonal L. Here $\alpha(\zeta)$, $\beta(\zeta)$, $g(\zeta)$ are real functions of the Hölder class on each of the arcs Γ_j, admitting discontinuities of the first kind at the points ζ_j. The conditions $\alpha^2(\zeta) + \beta^2(\zeta) \equiv 1$, $\beta(\zeta) \geq 0$, are satisfied on Γ_j.

Some interesting classes of surfaces (symmetric domes) and families of pairs of functions $(\alpha(\zeta), \beta(\zeta))$ are found. For these classes index $\varkappa$ of the boundary condition (1) is calculated by formula $\varkappa = -4 + \sum\limits_{k=0}^{n} f_k(\theta_k, \alpha, \beta)$ where f_k are piecewise constant integral valued functions taking values in $[-2; 3]$, θ_k is the value of the internal angle at the point ζ_k. Further, according to the scheme of [2], a criterion of quasi correctness of the main boundary value problem for such surfaces is obtained in the geometric form.

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MSC 39A14.

The Liuoville-type theorems for stationary Ginzburg-Landau equation on Lipshitz manifolds of a special type

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The paper is devoted to the problems of existence of positive solutions of the stationary Ginzburg-Landau equation on Lipschitz manifolds of a special type. Let us describe them in more detail.

We consider the Lipschitz manifold M_g that is isometric to the direct product $R_+ \times S$ (where $R_+ = (0, +\infty)$, and S is a compact Lipschitz manifold without boundary) with the metric:

$$ds^2 = dr^2 + g^2(r)d\theta^2$$

Here g is a Lipschitz continuous, nondecreasing positive function on R_+. Suppose also that $n = dim\, M_g$.

We consider the stationary case of the well-known Ginzburg-Landau equation on M_g

$$-\Delta u = c(x)f(u), \tag{1}$$

where $f(0) = f(a) = 0$ for some $a > 0$, $f(u) > 0$ on $(0, a)$ and $f(u) < 0$ on $(a, +\infty)$, $c(x)$ is a positive function.

As the solution of equation (1) on M_g we consider a Lipschitz continuous function u such that for any set $\Omega \subset M_g$ and for any positive function $\phi(x) \in C_0^1(\Omega)$ satisfies the equation

$$\int\limits_{\Omega} \nabla u \cdot \nabla \phi \, dx = \int\limits_{\Omega} c(x) f(u)\, \phi(x)\, dx,$$

where ∇u is the gradient of u.

Let $0 < c_1 < c(x) < c_2 < \infty$, $f(s)$ is a Lipschitz continuous on $[0, a]$. Then the following statement is true.

Theorem 1. *Let the following conditions be satisfied.*
1. There exists a constant $q > 1$ for which

$$\limsup_{\rho \to \infty} \rho^{\frac{2}{q-1}} \left(\frac{\int\limits_{\rho/4}^{2\rho} g^{n-1}(r)dr}{\int\limits_{\rho/2}^{\rho} g^{n-1}(r)dr} \right)^{\frac{1}{q-1}} \int\limits_{2\rho}^{\infty} \frac{ds}{g^{n-1}(r)} = +\infty.$$

2. There exist constants $\delta(q) > 0$ and $\sigma(q, \delta) > 0$ such that for all $s \in (0, \delta)$ the following holds: $f(s) \geqslant \sigma s^q$.

Then any solution of equation (1) that satisfy condition $0 \leqslant u \leqslant a$, is the identity constant.

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MSC 30C99

On the curvature of level-lines in the class of regular functions

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For any natural number n and for real constants A, B satisfying $-1 \leq A < B \leq 1$, let $P_n(A, B)$ [1] denote the class of regular in the unit disc $|z| < 1$ functions $p(z)$, $p(0) = 1$ defines by

$$p(z) = \frac{1 + Az^{n-1}\omega(z)}{1 + Bz^{n-1}\omega(z)},$$

where $\omega(z)$ is regular in the unit disc $|z| < 1$ and $\omega(0) = 0, |\omega(z)| < 1$.

We consider the subclass of $P_n(A, B)$:

$$P_b(n, A, B) = \{p(z) : p(z) \in P_n(A, B),\ p^{(n)}(0) = n!(B - A)b,\ 0 \leq b \leq 1\}.$$

Let $U_b(n, A, B)$ denote the class of regular in the unit disc $|z| < 1$ function $f(z)$ such as $f'(z) \in P_b(n, A, B)$.

In this paper we obtain the lower sharp estimated curvature

$$K_r = \frac{Re\left(1 + \frac{zf''(z)}{f'(z)}\right)}{|zf'(z)|}, z = re^{i\varphi},$$

of the image of circle $|z| = r > 0$ under the mapping $f(re^{i\varphi})$, where $f(z) \in U_b(n, A, B)$.

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Set of values of initial coefficients for symmetric mappings of the upper half-plane

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There are many results that determine value regions of initial coefficients for classes of univalent functions. In particular for the set of univalent functions in the unit disc see [1].

Let $\mathbb{H}$ be the upper half-plane. Denote H the set of all univalent, i.e. holomorphic and injective, mappings $f : \mathbb{H} \to \mathbb{H}$ with hydrodynamic normalization at infinity, i.e. $f(z) = z - \frac{c}{z} + \alpha(z)$, where $c \geq 0$ and α satisfies $\angle \lim_{z\to\infty} z \cdot \alpha(z) = 0$. For the class H and its subclasses, the estimates of the coefficients are given in [2, pp.242-252], see also [3]. We consider functions of class H mapping $\mathbb{H}$ onto domains having a bounded complement to the upper half-plane. The class of such functions is denoted by H_L and is characterized by the Laurent expansion near infinity

$$f(z) = z - \frac{c}{z} + \frac{c_2}{z^2} + \dots . \tag{1}$$

Denote $H_L^* = \{f : f \in H_L \quad \text{and} \quad f(-\overline{z}) = -\overline{f(z)} \quad \forall z \in \mathbb{H}\}$. The set H_L^* consists of all $f \in H_L$ such that the image $f(\mathbb{H})$ is symmetric with respect to the imaginary axis and implies that all even coefficients in the expansion (1) are equal to zero. Fix $c = T \geq 0$ and define $K_L^*(T) = \{(c_3(T); c_5(T)) : \ f \in H_L^*, \}$ where c, c_3 and c_5 are coefficients from expansion (1).

Theorem 1. *Fix $T > 0$. Let $x = c_3(T)$, $y = c_5(T)$. Define the three curves γ_1, γ_2 and γ_3 by*

$$\gamma_1 = \{x = -\frac{T^2}{2}, y < -\frac{T^3}{2}\},$$

$$\gamma_2 = \{-T^2 \leq x \leq -\frac{T^2}{2}, y = T \cdot x - \frac{2}{3}(-T^2 - 2x)^{3/2}\},$$

$$\gamma_3 = \{x \leq -T^2, y = \frac{T^3}{3} + T \cdot x - \frac{1}{T}x^2\}.$$

The closure $\overline{K_L^*(T)}$ of $K_L^*(T)$ is the set in the third coordinate quarter bounded by γ_1, γ_2 and γ_3 and $K_L^*(T) = \{(-\frac{T^2}{2}; -\frac{T^3}{2})\} \cup \overline{K_L^*(T)} \setminus \gamma_1$.

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